

and due to the charge  $Q_2 + q = \frac{Q_2 + q}{2A\epsilon_0}$  (upward).

The net electric field at  $P$  due to all the four charged surfaces is (in the downward direction)

$$\frac{Q_1 - q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{Q_2 + q}{2A\epsilon_0}$$

As the point  $P$  is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - Q_2 - q = 0$$

or,  $q = \frac{Q_1 - Q_2}{2}$  ... (i)

Thus,  $Q_1 - q = \frac{Q_1 + Q_2}{2}$  ... (ii)

and  $Q_2 + q = \frac{Q_1 + Q_2}{2}$

□

### QUESTIONS FOR SHORT ANSWER

1. A small plane area is rotated in an electric field. In which orientation of the area is the flux of electric field through the area maximum? In which orientation is it zero?
2. A circular ring of radius  $r$  made of a nonconducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through  $180^\circ$ . Does the flux of electric field change? If yes, does it decrease or increase?
3. A charge  $Q$  is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell? If a point charge is brought close to the shell, will the field at the centre change? Does your answer depend on whether the shell is conducting or nonconducting?
4. A spherical shell made of plastic, contains a charge  $Q$  distributed uniformly over its surface. What is the electric field inside the shell? If the shell is hammered to deshape it without altering the charge, will the field inside be changed? What happens if the shell is made of a metal?
5. A point charge  $q$  is placed in a cavity in a metal block. If a charge  $Q$  is brought outside the metal, will the charge  $q$  feel an electric force?
6. A rubber balloon is given a charge  $Q$  distributed uniformly over its surface. Is the field inside the balloon zero everywhere if the balloon does not have a spherical surface?
7. It is said that any charge given to a conductor comes to its surface. Should all the protons come to the surface? Should all the electrons come to the surface? Should all the free electrons come to the surface?

### OBJECTIVE I

1. A charge  $Q$  is uniformly distributed over a large plastic plate. The electric field at a point  $P$  close to the centre of the plate is  $10 \text{ V/m}$ . If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge  $Q$ , the electric field at the point  $P$  will become  
(a) zero (b)  $5 \text{ V/m}$  (c)  $10 \text{ V/m}$  (d)  $20 \text{ V/m}$ .
2. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be  
(a) towards the plate (b) away from the plate  
(c) parallel to the plate (d) zero.
3. A thin, metallic spherical shell contains a charge  $Q$  on it. A point charge  $q$  is placed at the centre of the shell and another charge  $q_1$  is placed outside it as shown in figure (30-Q1). All the three charges are positive. The

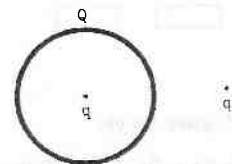


Figure 30-Q1

Using these equations, the distribution shown in the figure (30-W6) can be redrawn as in figure (30-W7).

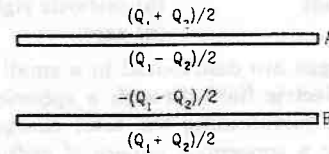


Figure 30-W7

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

force on the charge at the centre is

- (a) towards left (b) towards right  
(c) upward (d) zero.
4. Consider the situation of the previous problem. The force on the central charge due to the shell is  
(a) towards left (b) towards right  
(c) upward (d) zero.
5. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V-m. The flux over a concentric sphere of radius 20 cm will be  
(a) 25 V-m (b) 50 V-m (c) 100 V-m (d) 200 V-m.
6. Figure (30-Q2a) shows an imaginary cube of edge  $L/2$ . A uniformly charged rod of length  $L$  moves towards left at a small but constant speed  $v$ . At  $t = 0$ , the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in figure (30-Q2b) represents the flux of the electric field through the cube as the rod goes through it?

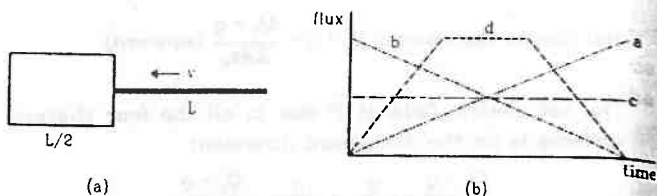


Figure 30-Q2

7. A charge  $q$  is placed at the centre of the open end of a cylindrical vessel (figure 30-Q3). The flux of the electric field through the surface of the vessel is  
(a) zero (b)  $q/\epsilon_0$  (c)  $q/2\epsilon_0$  (d)  $2q/\epsilon_0$ .

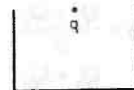


Figure 30-Q3

## OBJECTIVE II

1. Mark the correct options:  
(a) Gauss's law is valid only for symmetrical charge distributions.  
(b) Gauss's law is valid only for charges placed in vacuum.  
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.  
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.
2. A positive point charge  $Q$  is brought near an isolated metal cube.  
(a) The cube becomes negatively charged.  
(b) The cube becomes positively charged.  
(c) The interior becomes positively charged and the surface becomes negatively charged.  
(d) The interior remains charge free and the surface gets nonuniform charge distribution.
3. A large nonconducting sheet  $M$  is given a uniform charge density. Two uncharged small metal rods  $A$  and  $B$  are placed near the sheet as shown in figure (30-Q4).  
(a)  $M$  attracts  $A$ . (b)  $M$  attracts  $B$ .  
(c)  $A$  attracts  $B$ . (d)  $B$  attracts  $A$ .

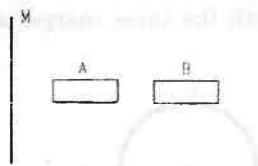


Figure 30-Q4

4. If the flux of the electric field through a closed surface is zero,

- (a) the electric field must be zero everywhere on the surface  
(b) the electric field may be zero everywhere on the surface  
(c) the charge inside the surface must be zero  
(d) the charge in the vicinity of the surface must be zero.
5. An electric dipole is placed at the centre of a sphere. Mark the correct options:  
(a) The flux of the electric field through the sphere is zero.  
(b) The electric field is zero at every point of the sphere.  
(c) The electric field is not zero anywhere on the sphere.  
(d) The electric field is zero on a circle on the sphere.
6. Figure (30-Q5) shows a charge  $q$  placed at the centre of a hemisphere. A second charge  $Q$  is placed at one of the positions  $A$ ,  $B$ ,  $C$  and  $D$ . In which position(s) of this second charge, the flux of the electric field through the hemisphere remains unchanged?  
(a)  $A$  (b)  $B$  (c)  $C$  (d)  $D$ .

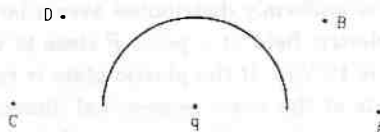


Figure 30-Q5

7. A closed surface  $S$  is constructed around a conducting wire connected to a battery and a switch (figure 30-Q6). As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface  $S$  is equal to the number of electrons leaving it. On closing

the switch, the flux of the electric field through the closed surface

- (a) is increased (b) is decreased  
(c) remains unchanged (d) remains zero.

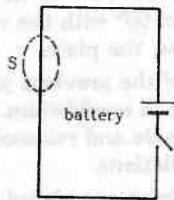


Figure 30-Q6

8. Figure (30-Q7) shows a closed surface which intersects a conducting sphere. If a positive charge is placed at

the point P, the flux of the electric field through the closed surface

- (a) will remain zero (b) will become positive  
(c) will become negative (d) will become undefined.

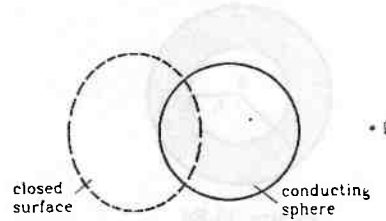


Figure 30-Q7

## EXERCISES

- The electric field in a region is given by  $\vec{E} = \frac{3}{5} E_0 \vec{i} + \frac{4}{5} E_0 \vec{j}$  with  $E_0 = 2.0 \times 10^3 \text{ N/C}$ . Find the flux of this field through a rectangular surface of area  $0.2 \text{ m}^2$  parallel to the  $Y-Z$  plane.
- A charge  $Q$  is uniformly distributed over a rod of length  $l$ . Consider a hypothetical cube of edge  $l$  with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.
- Show that there can be no net-charge in a region in which the electric field is uniform at all points.
- The electric field in a region is given by  $\vec{E} = \frac{E_0 x}{l} \vec{i}$ . Find the charge contained inside a cubical volume bounded by the surfaces  $x = 0$ ,  $x = a$ ,  $y = 0$ ,  $y = a$ ,  $z = 0$  and  $z = a$ . Take  $E_0 = 5 \times 10^3 \text{ N/C}$ ,  $l = 2 \text{ cm}$  and  $a = 1 \text{ cm}$ .
- A charge  $Q$  is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.
- A charge  $Q$  is placed at a distance  $a/2$  above the centre of a horizontal, square surface of edge  $a$  as shown in figure (30-E1). Find the flux of the electric field through the square surface.

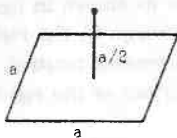


Figure 30-E1

- Find the flux of the electric field through a spherical surface of radius  $R$  due to a charge of  $10^{-6} \text{ C}$  at the centre and another equal charge at a point  $2R$  away from the centre (figure 30-E2).

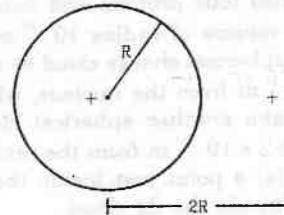


Figure 30-E2

- A charge  $Q$  is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure 30-E3).

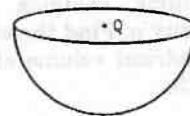


Figure 30-E3

- A spherical volume contains a uniformly distributed charge of density  $2.0 \times 10^{-4} \text{ C/m}^3$ . Find the electric field at a point inside the volume at a distance  $4.0 \text{ cm}$  from the centre.
- The radius of a gold nucleus ( $Z = 79$ ) is about  $7.0 \times 10^{-15} \text{ m}$ . Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at (a) the surface of the nucleus and (b) at the middle point of a radius. Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface?
- A charge  $Q$  is distributed uniformly within the material of a hollow sphere of inner and outer radii  $r_1$  and  $r_2$  (figure 30-E4). Find the electric field at a point  $P$  a

distance  $x$  away from the centre for  $r_1 < x < r_2$ . Draw a rough graph showing the electric field as a function of  $x$  for  $0 < x < 2r_2$  (figure 30-E4).

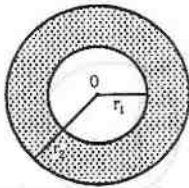


Figure 30-E4

12. A charge  $Q$  is placed at the centre of an uncharged, hollow metallic sphere of radius  $a$ . (a) Find the surface charge density on the inner surface and on the outer surface. (b) If a charge  $q$  is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces? (c) Find the electric field inside the sphere at a distance  $x$  from the centre in the situations (a) and (b).
13. Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius  $10^{-15}$  m. The two  $1s$  electrons make a spherical charge cloud at an average distance of  $1.3 \times 10^{-11}$  m from the nucleus, whereas the two  $2s$  electrons make another spherical charge cloud at an average distance of  $5.2 \times 10^{-11}$  m from the nucleus. Find the electric field at (a) a point just inside the  $1s$  cloud and (b) a point just inside the  $2s$  cloud.
14. Find the magnitude of the electric field at a point 4 cm away from a line charge of density  $2 \times 10^{-6}$  C/m.
15. A long cylindrical wire carries a positive charge of linear density  $2.0 \times 10^{-8}$  C/m. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.
16. A long cylindrical volume contains a uniformly distributed charge of density  $\rho$ . Find the electric field at a point  $P$  inside the cylindrical volume at a distance  $x$  from its axis (figure 30-E5)

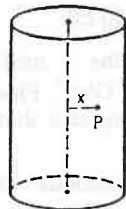


Figure 30-E5

17. A nonconducting sheet of large surface area and thickness  $d$  contains uniform charge distribution of density  $\rho$ . Find the electric field at a point  $P$  inside the plate, at a distance  $x$  from the central plane. Draw a qualitative graph of  $E$  against  $x$  for  $0 < x < d$ .
18. A charged particle having a charge of  $-2.0 \times 10^{-9}$  C is placed close to a nonconducting plate having a surface charge density  $4.0 \times 10^{-10}$  C/m<sup>2</sup>. Find the force of attraction between the particle and the plate.

19. One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of  $4.0 \times 10^{-9}$  C. In equilibrium, the thread makes an angle of  $60^\circ$  with the vertical. Find the surface charge density on the plate.
20. Consider the situation of the previous problem. (a) Find the tension in the string in equilibrium. (b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.
21. Two large conducting plates are placed parallel to each other with a separation of 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces.
22. Two large conducting plates are placed parallel to each other and they carry equal and opposite charges with surface density  $\sigma$  as shown in figure (30-E6). Find the electric field (a) at the left of the plates, (b) in between the plates and (c) at the right of the plates.

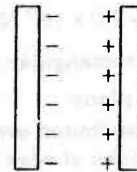


Figure 30-E6

23. Two conducting plates  $X$  and  $Y$ , each having large surface area  $A$  (on one side), are placed parallel to each other as shown in figure (30-E7). The plate  $X$  is given a charge  $Q$  whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate  $X$ , (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric field at a point to the right of the plates.



Figure 30-E7

21. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure (30-E8). The leftmost plate is given a charge  $Q$ , the rightmost a charge  $-2Q$  and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.

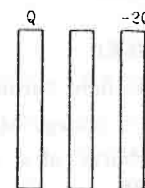


Figure 30-E8

## ANSWERS

## OBJECTIVE I

1. (c)    2. (a)    3. (d)    4. (b)    5. (a)    6. (d)  
7. (c)

## OBJECTIVE II

1. (d)    2. (d)    3. all  
4. (b), (c)    5. (a), (c)    6. (a), (c)  
7. (c), (d)    8. (b)

## EXERCISES

1.  $240 \frac{\text{N}\cdot\text{m}^2}{\text{C}}$   
2.  $Q/(2\epsilon_0)$   
4.  $2.2 \times 10^{-12} \text{ C}$   
5.  $Q/\epsilon_0$   
6.  $Q/(6\epsilon_0)$   
7.  $1.1 \times 10^4 \frac{\text{N}\cdot\text{m}^2}{\text{C}}$   
8.  $Q/(2\epsilon_0)$   
9.  $3.0 \times 10^{-7} \text{ N/C}$   
10. (a)  $2.32 \times 10^{21} \text{ N/C}$  (b)  $1.16 \times 10^{21} \text{ N/C}$

$$11. \frac{Q(x^2 - r_1^2)}{4\pi\epsilon_0 x^2 (r_2^3 - r_1^3)}$$

$$12. (a) -\frac{Q}{4\pi a^2}, \frac{Q}{4\pi a^2} \quad (b) -\frac{Q}{4\pi a^2}, \frac{Q+q}{4\pi a^2}$$

$$(c) \frac{Q}{4\pi\epsilon_0 x^2} \text{ in both situations}$$

13. (a)  $3.4 \times 10^{13} \text{ N/C}$     (b)  $1.1 \times 10^{12} \text{ N/C}$   
14.  $9 \times 10^5 \text{ N/C}$   
15.  $2.88 \times 10^{-17} \text{ J}$   
16.  $\rho x/(2\epsilon_0)$   
17.  $\rho x/\epsilon_0$   
18.  $0.45 \text{ N}$   
19.  $7.5 \times 10^{-7} \text{ C/m}^2$   
20. (a)  $0.20 \text{ N}$  (b)  $0.45 \text{ s}$   
21.  $0.505 \times 10^{-12} \text{ C/m}^2$   
22. (a) zero    (b)  $\sigma/\epsilon_0$     (c) zero  
23. (a)  $\frac{Q}{2A}$     (b)  $\frac{Q}{2A\epsilon_0}$  towards left    (c)  $\frac{Q}{2A\epsilon_0}$  towards right  
(d)  $\frac{Q}{2A\epsilon_0}$  towards right  
24.  $-Q/2$

## CHAPTER – 30 GAUSS'S LAW

1. Given :  $\vec{E} = 3/5 E_0 \hat{i} + 4/5 E_0 \hat{j}$

$E_0 = 2.0 \times 10^3$  N/C The plane is parallel to yz-plane.

Hence only  $3/5 E_0 \hat{i}$  passes perpendicular to the plane whereas  $4/5 E_0 \hat{j}$  goes parallel. Area =  $0.2 \text{ m}^2$  (given)

$$\therefore \text{Flux} = \vec{E} \cdot \vec{A} = 3/5 \times 2 \times 10^3 \times 0.2 = 2.4 \times 10^2 \text{ Nm}^2/\text{c} = 240 \text{ Nm}^2/\text{c}$$

2. Given length of rod = edge of cube =  $\ell$

Portion of rod inside the cube =  $\ell/2$

Total charge = Q.

Linear charge density =  $\lambda = Q/\ell$  of rod.

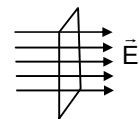
We know: Flux  $\propto$  charge enclosed.

Charge enclosed in the rod inside the cube.

$$= \ell/2 \epsilon_0 \times Q/\ell = Q/2 \epsilon_0$$

3. As the electric field is uniform.

Considering a perpendicular plane to it, we find that it is an equipotential surface. Hence there is no net current flow on that surface. Thus, net charge in that region is zero.



4. Given:  $E = \frac{E_0 \lambda}{\ell} \hat{i}$   $\ell = 2$  cm,  $a = 1$  cm.

$E_0 = 5 \times 10^3$  N/C. From fig. We see that flux passes mainly through surface areas. ABDC & EFGH. As the AEFB & CHGD are parallel to the Flux. Again in ABDC  $a = 0$ ; hence the Flux only passes through the surface are EFGH.

$$E = \frac{E_c \lambda}{\ell} \hat{i}$$

$$\text{Flux} = \frac{E_0 \lambda}{L} \times \text{Area} = \frac{5 \times 10^3 \times a}{\ell} \times a^2 = \frac{5 \times 10^3 \times a^3}{\ell} = \frac{5 \times 10^3 \times (0.01)^3}{2 \times 10^{-2}} = 2.5 \times 10^{-1}$$

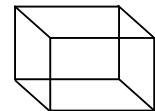
$$\text{Flux} = \frac{q}{\epsilon_0} \text{ so, } q = \epsilon_0 \times \text{Flux}$$

$$= 8.85 \times 10^{-12} \times 2.5 \times 10^{-1} = 2.2125 \times 10^{-12} \text{ c}$$

5. According to Gauss's Law Flux =  $\frac{q}{\epsilon_0}$

Since the charge is placed at the centre of the cube. Hence the flux passing through the

$$\text{six surfaces} = \frac{Q}{6\epsilon_0} \times 6 = \frac{Q}{\epsilon_0}$$



6. Given – A charge is placed o a plain surface with area =  $a^2$ , about  $a/2$  from its centre.

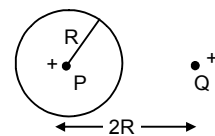
Assumption : let us assume that the given plain forms a surface of an imaginary cube. Then the charge is found to be at the centre of the cube.

$$\text{Hence flux through the surface} = \frac{Q}{\epsilon_0} \times \frac{1}{6} = \frac{Q}{6\epsilon_0}$$

7. Given: Magnitude of the two charges placed =  $10^{-7}$  c.

We know: from Gauss's law that the flux experienced by the sphere is only due to the internal charge and not by the external one.

$$\text{Now } \oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0} = \frac{10^{-7}}{8.85 \times 10^{-12}} = 1.1 \times 10^4 \text{ N-m}^2/\text{C}.$$



8. We know: For a spherical surface

$$\text{Flux} = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \quad [\text{by Gauss law}]$$

$$\text{Hence for a hemisphere} = \text{total surface area} = \frac{q}{\epsilon_0} \times \frac{1}{2} = \frac{q}{2\epsilon_0}$$



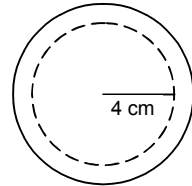
9. Given: Volume charge density =  $2.0 \times 10^{-4} \text{ C/m}^3$

In order to find the electric field at a point  $4 \text{ cm} = 4 \times 10^{-2} \text{ m}$  from the centre let us assume a concentric spherical surface inside the sphere.

$$\text{Now, } \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\text{But } \sigma = \frac{q}{4/3\pi R^3} \quad \text{so, } q = \sigma \times 4/3 \pi R^3$$

$$\begin{aligned} \text{Hence} &= \frac{\sigma \times 4/3 \times 22/7 \times (4 \times 10^{-2})^3}{\epsilon_0} \times \frac{1}{4 \times 22/7 \times (4 \times 10^{-2})^2} \\ &= 2.0 \times 10^{-4} \times \frac{1}{3} \times 4 \times 10^{-2} \times \frac{1}{8.85 \times 10^{-12}} = 3.0 \times 10^5 \text{ N/C} \end{aligned}$$



10. Charge present in a gold nucleus =  $79 \times 1.6 \times 10^{-19} \text{ C}$

Since the surface encloses all the charges we have:

$$(a) \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} = \frac{79 \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}}$$

$$\begin{aligned} E &= \frac{q}{\epsilon_0 ds} = \frac{79 \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}} \times \frac{1}{4 \times 3.14 \times (7 \times 10^{-15})^2} \quad [ \because \text{area} = 4\pi r^2 ] \\ &= 2.3195131 \times 10^{21} \text{ N/C} \end{aligned}$$

(b) For the middle part of the radius. Now here  $r = 7/2 \times 10^{-15} \text{ m}$

$$\text{Volume} = 4/3 \pi r^3 = \frac{48}{3} \times \frac{22}{7} \times \frac{343}{8} \times 10^{-45}$$

Charge enclosed =  $\zeta \times \text{volume}$  [  $\zeta$  : volume charge density ]

$$\text{But } \zeta = \frac{\text{Net charge}}{\text{Net volume}} = \frac{7.9 \times 1.6 \times 10^{-19} \text{ C}}{\left(\frac{4}{3}\right) \times \pi \times 343 \times 10^{-45}}$$

$$\text{Net charged enclosed} = \frac{7.9 \times 1.6 \times 10^{-19}}{\left(\frac{4}{3}\right) \times \pi \times 343 \times 10^{-45}} \times \frac{4}{3} \pi \times \frac{343}{8} \times 10^{-45} = \frac{7.9 \times 1.6 \times 10^{-19}}{8}$$

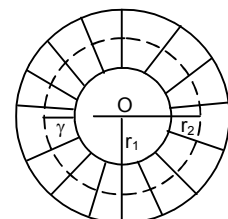
$$\oint \vec{E} \cdot d\vec{s} = \frac{q \text{ enclosed}}{\epsilon_0}$$

$$\Rightarrow E = \frac{7.9 \times 1.6 \times 10^{-19}}{8 \times \epsilon_0 \times S} = \frac{7.9 \times 1.6 \times 10^{-19}}{8 \times 8.85 \times 10^{-12} \times 4\pi \times \frac{49}{4} \times 10^{-30}} = 1.159 \times 10^{21} \text{ N/C}$$

11. Now, Volume charge density =  $\frac{Q}{\frac{4}{3} \pi \times (r_2^3 - r_1^3)}$

$$\therefore \zeta = \frac{3Q}{4\pi(r_2^3 - r_1^3)}$$

$$\text{Again volume of sphere having radius } x = \frac{4}{3} \pi x^3$$



Now charge enclosed by the sphere having radius

$$\chi = \left( \frac{4}{3} \pi \chi^3 - \frac{4}{3} \pi r_1^3 \right) \times \frac{Q}{\frac{4}{3} \pi r_2^3 - \frac{4}{3} \pi r_1^3} = Q \left( \frac{\chi^3 - r_1^3}{r_2^3 - r_1^3} \right)$$

Applying Gauss's law  $- E \times 4\pi\chi^2 = \frac{q \text{ enclosed}}{\epsilon_0}$

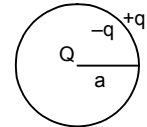
$$\Rightarrow E = \frac{Q}{\epsilon_0} \left( \frac{\chi^3 - r_1^3}{r_2^3 - r_1^3} \right) \times \frac{1}{4\pi\chi^2} = \frac{Q}{4\pi\epsilon_0\chi^2} \left( \frac{\chi^3 - r_1^3}{r_2^3 - r_1^3} \right)$$

12. Given: The sphere is uncharged metallic sphere.

Due to induction the charge induced at the inner surface =  $-Q$ , and that outer surface =  $+Q$ .

(a) Hence the surface charge density at inner and outer surfaces =  $\frac{\text{charge}}{\text{total surface area}}$

$$= -\frac{Q}{4\pi a^2} \text{ and } \frac{Q}{4\pi a^2} \text{ respectively.}$$



(b) Again if another charge 'q' is added to the surface. We have inner surface charge density =  $-\frac{Q}{4\pi a^2}$ ,

because the added charge does not affect it.

On the other hand the external surface charge density =  $Q + \frac{q}{4\pi a^2}$  as the 'q' gets added up.

(c) For electric field let us assume an imaginary surface area inside the sphere at a distance 'x' from centre. This is same in both the cases as the 'q' is ineffective.

$$\text{Now, } \oint E \cdot ds = \frac{Q}{\epsilon_0} \text{ So, } E = \frac{Q}{\epsilon_0} \times \frac{1}{4\pi x^2} = \frac{Q}{4\pi\epsilon_0 x^2}$$

13. (a) Let the three orbits be considered as three concentric spheres A, B & C.

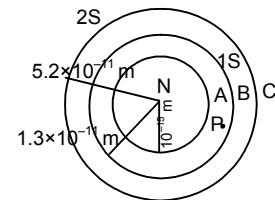
Now, Charge of 'A' =  $4 \times 1.6 \times 10^{-16} \text{ C}$

Charge of 'B' =  $2 \times 1.6 \times 10^{-16} \text{ C}$

Charge of 'C' =  $2 \times 1.6 \times 10^{-16} \text{ C}$

As the point 'P' is just inside 1s, so its distance from centre =  $1.3 \times 10^{-11} \text{ m}$

$$\text{Electric field} = \frac{Q}{4\pi\epsilon_0 x^2} = \frac{4 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (1.3 \times 10^{-11})^2} = 3.4 \times 10^{13} \text{ N/C}$$



(b) For a point just inside the 2 s cloud

Total charge enclosed =  $4 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19} = 2 \times 1.6 \times 10^{-19}$

Hence, Electric field,

$$\vec{E} = \frac{2 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (5.2 \times 10^{-11})^2} = 1.065 \times 10^{12} \text{ N/C} \approx 1.1 \times 10^{12} \text{ N/C}$$

14. Drawing an electric field around the line charge we find a cylinder of radius  $4 \times 10^{-2} \text{ m}$ .

Given:  $\lambda$  = linear charge density

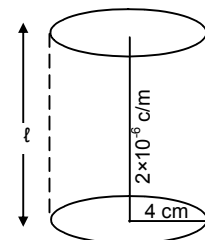
Let the length be  $\ell = 2 \times 10^{-6} \text{ c/m}$

$$\text{We know } \oint E \cdot dl = \frac{Q}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

$$\Rightarrow E \times 2\pi r \ell = \frac{\lambda \ell}{\epsilon_0} \Rightarrow E = \frac{\lambda}{\epsilon_0 \times 2\pi r}$$

For,  $r = 2 \times 10^{-2} \text{ m}$  &  $\lambda = 2 \times 10^{-6} \text{ c/m}$

$$\Rightarrow E = \frac{2 \times 10^{-6}}{8.85 \times 10^{-12} \times 2 \times 3.14 \times 2 \times 10^{-2}} = 8.99 \times 10^5 \text{ N/C} \approx 9 \times 10^5 \text{ N/C}$$





15. Given :

$$\lambda = 2 \times 10^{-6} \text{ C/m}$$

For the previous problem.

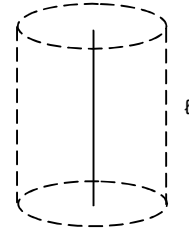
$$E = \frac{\lambda}{\epsilon_0 2\pi r} \text{ for a cylindrical electric field.}$$

Now, For experienced by the electron due to the electric field in wire = centripetal force.

$$Eq = mv^2 \quad \left[ \begin{array}{l} \text{we know, } m_e = 9.1 \times 10^{-31} \text{ kg,} \\ v_e = ?, r = \text{assumed radius} \end{array} \right]$$

$$\Rightarrow \frac{1}{2} Eq = \frac{1}{2} \frac{mv^2}{r}$$

$$\Rightarrow KE = 1/2 \times E \times q \times r = \frac{1}{2} \times \frac{\lambda}{\epsilon_0 2\pi r} \times 1.6 \times 10^{-19} = 2.88 \times 10^{-17} \text{ J.}$$



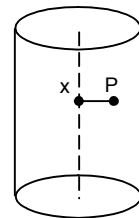
16. Given: Volume charge density =  $\zeta$

Let the height of cylinder be h.

$$\therefore \text{Charge } Q \text{ at } P = \zeta \times 4\pi r^2 \times h$$

$$\text{For electric field } \oint E \cdot ds = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 \times ds} = \frac{\zeta \times 4\pi r^2 \times h}{\epsilon_0 \times 2 \times \pi \times r \times h} = \frac{2\zeta r}{\epsilon_0}$$



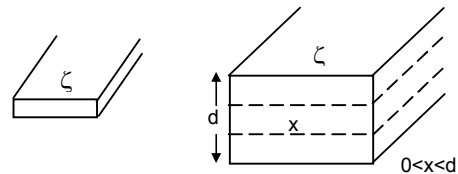
17.  $\oint E \cdot dA = \frac{Q}{\epsilon_0}$

Let the area be A.

Uniform charge distribution density is  $\zeta$

$$Q = \zeta A$$

$$E = \frac{Q}{\epsilon_0} \times dA = \frac{\zeta \times a \times \chi}{\epsilon_0 \times A} = \frac{\zeta \chi}{\epsilon_0}$$



18.  $Q = -2.0 \times 10^{-6} \text{ C}$  Surface charge density =  $4 \times 10^{-6} \text{ C/m}^2$

$$\text{We know } \vec{E} \text{ due to a charge conducting sheet} = \frac{\sigma}{2\epsilon_0}$$

Again Force of attraction between particle & plate

$$= Eq = \frac{\sigma}{2\epsilon_0} \times q = \frac{4 \times 10^{-6} \times 2 \times 10^{-6}}{2 \times 8 \times 10^{-12}} = 0.452 \text{ N}$$

19. Ball mass = 10g

$$\text{Charge} = 4 \times 10^{-6} \text{ C}$$

Thread length = 10 cm

Now from the fig,  $T \cos \theta = mg$

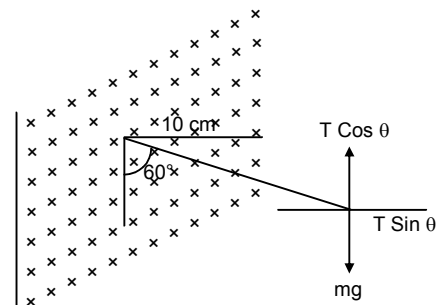
$T \sin \theta = \text{electric force}$

$$\text{Electric force} = \frac{\sigma q}{2\epsilon_0} \quad (\sigma \text{ surface charge density})$$

$$T \sin \theta = \frac{\sigma q}{2\epsilon_0}, \quad T \cos \theta = mg$$

$$\tan \theta = \frac{\sigma q}{2mg\epsilon_0}$$

$$\sigma = \frac{2mg\epsilon_0 \tan \theta}{q} = \frac{2 \times 8.85 \times 10^{-12} \times 10 \times 10^{-3} \times 9.8 \times 1.732}{4 \times 10^{-6}} = 7.5 \times 10^{-7} \text{ C/m}^2$$



20. (a) Tension in the string in Equilibrium

$$T \cos 60^\circ = mg$$

$$\Rightarrow T = \frac{mg}{\cos 60^\circ} = \frac{10 \times 10^{-3} \times 10}{1/2} = 10^{-1} \times 2 = 0.20 \text{ N}$$

(b) Straingtening the same figure.

Now the resultant for 'R'

Induces the acceleration in the pendulum.

$$T = 2 \times \pi \sqrt{\frac{\ell}{g}} = 2\pi \sqrt{\frac{\ell}{\left[ g^2 + \left( \frac{\sigma q}{2\epsilon_0 m} \right)^2 \right]^{1/2}}} = 2\pi \sqrt{\frac{\ell}{\left[ 100 + \left( 0.2 \times \frac{\sqrt{3}}{2 \times 10^{-2}} \right)^2 \right]^{1/2}}}$$

$$= 2\pi \sqrt{\frac{\ell}{(100 + 300)^{1/2}}} = 2\pi \sqrt{\frac{\ell}{20}} = 2 \times 3.1416 \times \sqrt{\frac{10 \times 10^{-2}}{20}} = 0.45 \text{ sec.}$$

21.  $s = 2\text{cm} = 2 \times 10^{-2}\text{m}$ ,  $u = 0$ ,  $a = ?$   $t = 2\mu\text{s} = 2 \times 10^{-6}\text{s}$

Acceleration of the electron,  $s = (1/2) at^2$

$$2 \times 10^{-2} = (1/2) \times a \times (2 \times 10^{-6})^2 \Rightarrow a = \frac{2 \times 2 \times 10^{-2}}{4 \times 10^{-12}} \Rightarrow a = 10^{10} \text{ m/s}^2$$

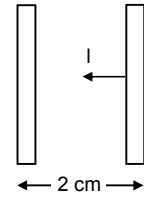
The electric field due to charge plate =  $\frac{\sigma}{\epsilon_0}$

Now, electric force =  $\frac{\sigma}{\epsilon_0} \times q = \text{acceleration} = \frac{\sigma}{\epsilon_0} \times \frac{q}{m_e}$

$$\text{Now } \frac{\sigma}{\epsilon_0} \times \frac{q}{m_e} = 10^{10}$$

$$\Rightarrow \sigma = \frac{10^{10} \times \epsilon_0 \times m_e}{q} = \frac{10^{10} \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$$

$$= 50.334 \times 10^{-14} = 0.50334 \times 10^{-12} \text{ C/m}^2$$



22. Given: Surface density =  $\sigma$

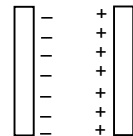
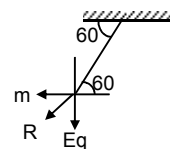
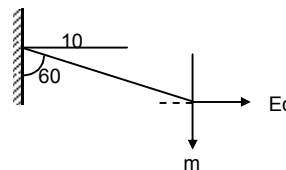
(a) & (c) For any point to the left & right of the dual plater, the electric field is zero.

As there are no electric flux outside the system.

(b) For a test charge put in the middle.

It experiences a fore  $\frac{\sigma q}{2\epsilon_0}$  towards the (-ve) plate.

Hence net electric field  $\frac{1}{q} \left( \frac{\sigma q}{2\epsilon_0} + \frac{\sigma q}{2\epsilon_0} \right) = \frac{\sigma}{\epsilon_0}$



23. (a) For the surface charge density of a single plate.

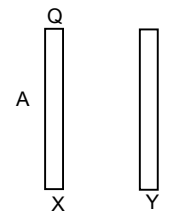
Let the surface charge density at both sides be  $\sigma_1$  &  $\sigma_2$

$$\sigma_1 \sigma_2 = \frac{\sigma_1}{2\epsilon_0} \& \frac{\sigma_2}{2\epsilon_0}$$

Due to a net balanced electric field on the plate  $\frac{\sigma_1}{2\epsilon_0} \& \frac{\sigma_2}{2\epsilon_0}$

$$\therefore \sigma_1 = \sigma_2 \text{ So, } q_1 = q_2 = Q/2$$

$$\therefore \text{Net surface charge density} = Q/2A$$



(b) Electric field to the left of the plates =  $\frac{\sigma}{\epsilon_0}$

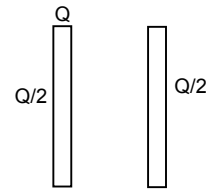
Since  $\sigma = Q/2A$  Hence Electricfield =  $Q/2A\epsilon_0$

This must be directed toward left as 'X' is the charged plate.

(c) & (d) Here in both the cases the charged plate 'X' acts as the only source of electric field, with (+ve) in the inner side and 'Y' attracts towards it with (-ve) he in

its inner side. So for the middle portion  $E = \frac{Q}{2A\epsilon_0}$  towards right.

(d) Similarly for extreme right the outside of the 'Y' plate acts as positive and hence it repels to the right with  $E = \frac{Q}{2A\epsilon_0}$



24. Consider the Gaussian surface the induced charge be as shown in figure.

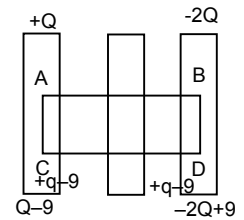
The net field at P due to all the charges is Zero.

$$\therefore -2Q + 9/2A \epsilon_0 (\text{left}) + 9/2A \epsilon_0 (\text{left}) + 9/2A \epsilon_0 (\text{right}) + Q - 9/2A \epsilon_0 (\text{right}) = 0$$

$$\Rightarrow -2Q + 9 - Q + 9 = 0 \Rightarrow 9 = 3/2 Q$$

$\therefore$  charge on the right side of right most plate

$$= -2Q + 9 = -2Q + 3/2 Q = -Q/2$$



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