Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-2013-P2-Model

Max.Marks:180

PHYSICS

Complete Ray Optics of JEE ADV syllabus (50%) + Vernier callipers, Screw gauge (50%) (excluding problems involving relative motion and calculus)

CHEMISTRY

Methods of expressing concentration of a solution - % by weight, Molarity, Molality, Normality, Mole fraction, ppm, % labelling of oleum, volume strength of hydrogen peroxide stoichiometry-II :Titrations, Volumetric analysis: neutralisation titrations - simple titrations, double titrations and back titrations, Redox titrations: oxalic-acid vs KMnO4, Mohr's salt vs KMnO4 ; iodometry , iodimetry, Degree of hardness of water (60%)

Mole , significant figures, laws of chemical combination, Chemical calculations based upon weight, volume relations of chemical equations, percentage composition of mixtures, empirical and molecular formula, Concept of redox reactions - oxidation number - Types of redox reactions, Balancing Redox reactions, Equivalent weight, (30%)

Cumulative syllabus (10%)

MATHS

Properties of AP,AM,GP,GM; Sum of 'n' Terms of AP & GP, Properties of HP & HM; Sum of infinite GP & AGP, method of differences/Vn method (60%); Triangular Inequality, AM-GM-HM Inequalities, Cauchy-Schwartz Inequality (30%); Cumulative (10%)

IIT-JEE-2013-P2-Model

Time: 07:30 AM to 10:30 AM **IMPORTANT INSTRUCTIONS** **Max Marks: 180**

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 8)	Questions with Multiple Correct Choice	3	-1	8	24
Sec – II(Q.N : 9 – 16)	Questions with Comprehension Type (4 Comprehensions – 2 +2+2+2 = 8Q)	3	-1	8	24
Sec – III(Q.N : 17 – 20)	Matrix Matching Type	3	-1	4	12
Total				**20**	**60**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 –28)	Questions with Multiple Correct Choice	3	-1	8	24
Sec – II(Q.N : 29 – 36)	Questions with Comprehension Type (4 Comprehensions – 2 +2+2+2 = 8Q)	3	-1	8	24
Sec – III(Q.N : 37 – 40)	Matrix Matching Type	3	-1	4	12
Total				**20**	**60**

MATHEMATICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 41 – 48)	Questions with Multiple Correct Choice	3	-1	8	24
Sec – II(Q.N : 49 – 56)	Questions with Comprehension Type (4 Comprehensions – 2 +2+2+2 = 8Q)	3	-1	8	24
Sec – III(Q.N : 57 – 60)	Matrix Matching Type	3	-1	4	12
Total				**20**	**60**

PHYSICS: **Max. Marks: 60**

SECTION – I

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

1. Choose the correct statement (/s) for zero error and zero correction.

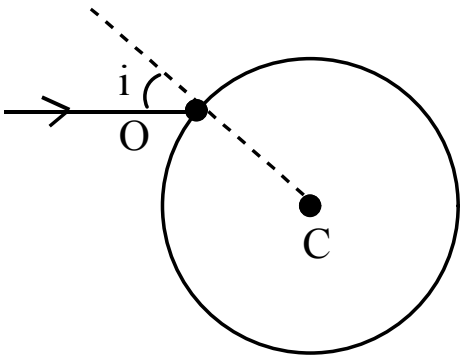
   A) If the zero of the vernier scale does not coincide with the zero of the main scale then the instrument is said to have a zero error.

   B) Zero correction has a magnitude equal to zero error but sign is opposite to that of the zero error.

   C) Zero error is positive when the zero of vernier scale lies to the left of the zero of the main scale.

   D) Zero error is negative when the zero of vernier scale lies to the left of the zero of the main scale.

2. A spherical transparent medium of refractive index n is placed in air. A thin parallel beam of light of monochromatic wavelength λ is incident at a point O of the sphere at an angle of incidence i (see figure). Then which of the following statements will be true for this optic system?



A) For the ray to come out of the sphere after refraction through it, the incident angle i should be less than the critical angle C for the medium of the sphere.

B) The angle of emergence θ (made with the normal) for the ray will be equal to the incident angle i for all values of i.

C) If angle $i \simeq 90^0$ (grazing incident) the emergent ray will come out grazing the surface.

D) If the angle of incident i is such that the angle of refraction at O is equal to the critical angle C for the medium of sphere, the emergent ray will be parallel to the incident ray

3. Choose the correct statements:

A) The least count of vernier calipers is the length of the smallest unit on the main scale

B) The least count of vernier calipers is the length of the smallest unit on the vernier scale

C) The vernier constant is the least count of the vernier calipers

D) Vernier calipers can't measure the depth of a cylinder

4. In a prism of angle A, critical angle θ_c and refractive index μ, the maximum deviation occurs when

A) the angle of incidence is $90°$

B) the angle of incidence may be is $\sin^{-1}\left[\left(\sqrt{\mu^2-1}\right)\sin A-\cos A\right]$

C) the angle of emergence is $\sin^{-1}\left[\left(\mu\sin\left(A-\theta_c\right)\right)\right]$

D) the angle of emergence is equal to the angle of incidence

5. Points $A(0,1)\ and\ B(12,5)$ are object –image pair (one of the point acts as object and the other point as image) x-axis is the principal axis of the mirror. This object image pair is

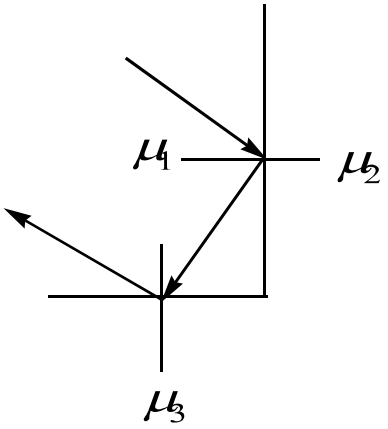
A) Due to convex mirror of focal length 2.5 units

B) Due to concave mirror having its pole at (2,0)

C) Real virtual pair

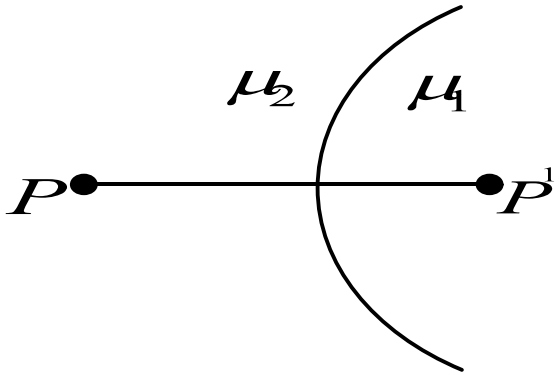
D) Due to concave mirror of focal length 2.5 units

6. In the diagram shown, a ray of light is incident on the inter face between 1 and 2 at an angle slightly greater than critical angle. The light suffers total internal reflection at this interface. After that the light ray falls at the interface of 1 and 3, and again is suffers total internal refraction. Which of the following relations hold true?



A) $\mu_1 < \mu_2 < \mu_3$ B) $\mu_1^2 - \mu_2^2 > \mu_3^2$ C) $\mu_1^2 - \mu_3^2 > \mu_2^2$ D) $\mu_1^2 + \mu_2^2 > \mu_3^2$

7. Two refracting media are separated by a spherical interface as shown in figure. pp' is the principal axis $\mu_1 \text{ and } \mu_2$ are the refractive indices of medium of incidence and medium of refraction respectively . if



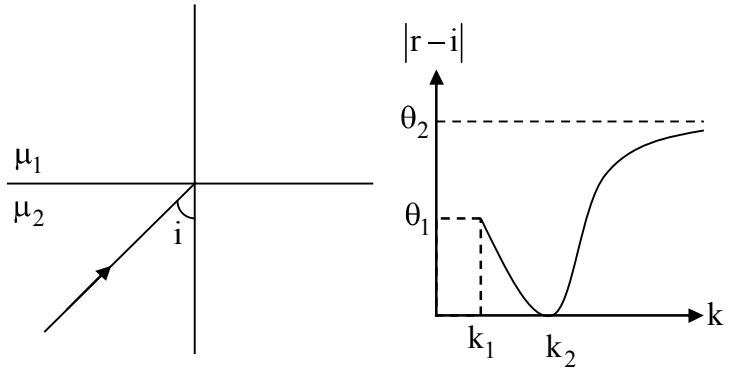
A) $\mu_2 > \mu_1$, There cannot be a real image of real object.

B) $\mu_2 > \mu_1$, There cannot be a real image of virtual object.

C) $\mu_1 > \mu_2$, There cannot be a virtual image of virtual object

D) $\mu_1 > \mu_2$, There cannot be a real image of real object

8. Let μ_1 and μ_2 be the refractive indices of media so that $k = \frac{\mu_1}{\mu_2}$, 'i'be the angle of incident and 'r'be the angle of refraction. Keeping $i = \frac{\pi}{3}$ constant, a graph 'k' versus '$|r-i|$'is drawn a shown bellow then:



A) The value of k_1 is $\frac{2}{\sqrt{3}}$

B) The value of $\theta_1 = \pi/6$

C) The value of $\theta_2 = \pi/3$

D) The value of k_2 is 1

SECTION - II
(COMPREHENSION TYPE)

This section contains **4 groups of questions.** Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.
Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

Paragraph for Questions 9 and 10:

In general vernier calipers can measure accurately up to 0.01 cm and for greater accuracy micrometer screw devices e.g., screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by tunning it axially. In one such type of screw gauge, the linear distance moved by the screw is 2mm in four rotations and there are 50 divisions on its cap. When nothing is put between its jaws, 20^{th} divisions of circular scale coincides with reference line, with zero of circular scale lying above the reference line. When a plate is placed between the jaws main scale reads 4 divisions and circular scale reads 20 divisions

9. The least count of screw gauge is

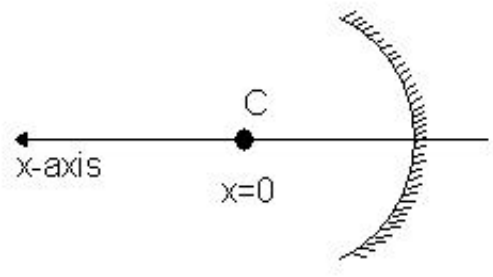
A) 0.1 mm B) 0.01 mm C) 0.05 mm D) 0.5 mm

10. Measurement corresponding to Zero error in the instrument is

A) $-0.02\ mm$ B) $+0.02\ mm$ C) $-0.3\ mm$ D) $+0.2\ mm$

Paragraph for Question 11 and 12:

A ball swings back and forth in front of a concave mirror. The motion of the ball is described approximately by the equation $x = f\cos\omega t$, where f is the focal length of the mirror and x is measured along the axis of mirror. The origin is taken at the centre of curvature of the mirror.



11. The distance of the image of the swinging ball from the mirror can be represented as:

A) $\left(\frac{2+\cos\omega t}{1-\cos\omega t}\right)f$ B) $\left(\frac{2-\cos\omega t}{1+\cos\omega t}\right)f$ C) $\left(\frac{2+\cos\omega t}{1+\cos\omega t}\right)f$ D) $\left(\frac{2-\cos\omega t}{1-\cos\omega t}\right)f$

12. The point where the ball appear to coincide with its image is

A) $x=-\frac{f}{3}$ B) $x=+\frac{f}{3}$ C) $x=\frac{f}{2}$ D) $x=0$

Paragraph for Questions 13 and 14:

The pitch of a screw gauge is 1 mm and there are 50 divisions on its circular scale. When nothing is put between the jaws the zero of circular lies 3 divisions below the reference line.

13. The least count of screw gauge is

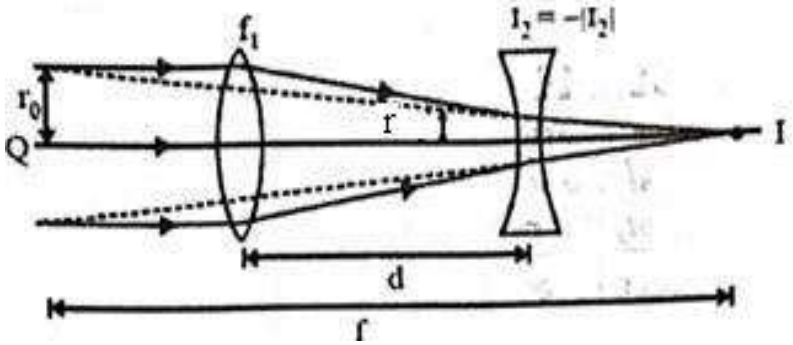
A) 0.002 mm B) 0.02 mm C) 0.02 cm D) 0.002 cm

14. Zero error in the instrument is

A) -0.06 mm B) 0.6 mm C) 0.06 mm D) none

Paragraph for Questions 15 and 16:

The following figure shows a simple version of a zoom. The converging lens has a focal length f_1 and the diverging lens has focal length $f_2 = -|f_2|$. The two lenses are separated by a variable distance d that is always less than f_1, also the magnitude of the focal length of the diverging lens satisfies the inequality $|f_2| > (f_1 - d)$



If the rays that emerge from the diverging lens and reach the final image point are extended backward to the left of the diverging lens, they will eventually expand to the original radius r_0 at the same point Q. To determine the effective focal length of the combination lens consider a bundle of parallel rays of radius r_0 entering the emerging lens.

15. At the point where ray enters the diverging lens, the radius of the ray bundle decreases to

A) $r = \left(\frac{f_1 - d}{f_1}\right) r_0$ B) $r = \left(\frac{f_1 - d}{f_1}\right) r_0$ C) $r = \left(\frac{f_1 - f_2}{f_1}\right) r_0$ D) $r = \frac{(d - f_1) f_2}{f_1 - f_2 - d}$

16. To the right of the diverging lens the final image I' is formed at a distance given by

A) $\frac{(f_1 - f_2) d}{f_1 - f_2 + d}$ B) $\frac{(f_1 - d) f_2}{f_1 - d + f_2}$ C) $\frac{f_1 - f_2 + d}{f_1 - f_2}$ D) $\frac{(d - f_1) f_2}{f_1 - f_2 - d}$

SECTION – III

(MATRIX MATCH TYPE)

This section contains **4 multiple choice questions.** Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

17.

Column I	Column II
(A) Diverging lens	(P) Focal length does not change on dipping in Water
(B)Converging lens	(Q) Always forms a virtual, erect and diminished image of a real object
(C)Concave mirror	(R) Can form virtual, erect and magnified image of a real object
(D)Convex mirror	(S) Can form real, inverted and diminished image of a real object
	(T) Focal length changes on dipping in water

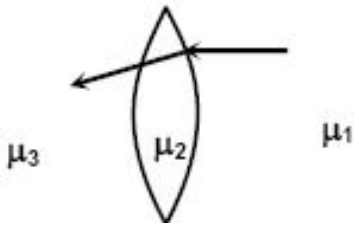
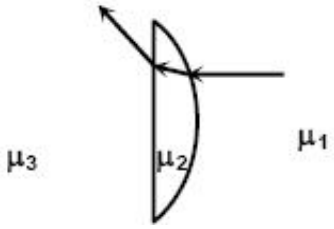
A) A– RST, B– PRS, C –PQ, D –QT

B) A – QT, B– RST, C –PQ, D –PRS

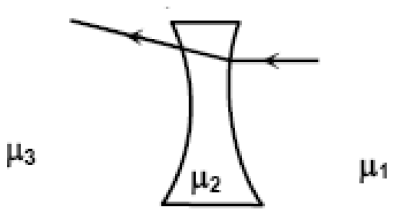
C) A – QT, B– RST, C –PRS, D –PQ

D) A– RST, B– QT, C –PRS, D –PQ

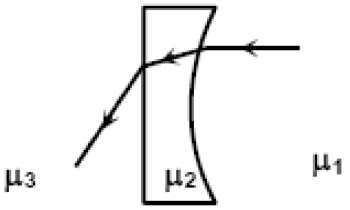
18. Two transparent media of refractive indices μ_1 and μ_2 have a solid lens shaped transparent material of refractive index μ_2 between them as show in figures in Column – II. A ray traversing these media is also shown in the figures. In Column – I different relationship between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column – II

Column I	**Column II**
(A) $\mu_1 < \mu_2$	(P) 
(B) $\mu_1 > \mu_2$	(Q) 

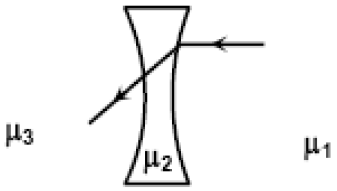
(C) $\mu_2 = \mu_3$ (R)



(D) $\mu_2 > \mu_3$ (S)



(T)



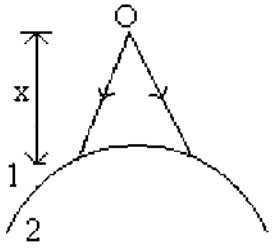
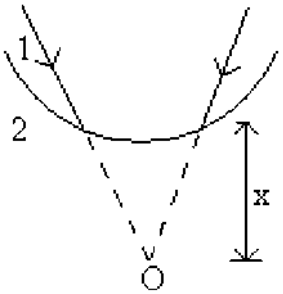
A) A – PR, B–QST, C–QS, D –PRT

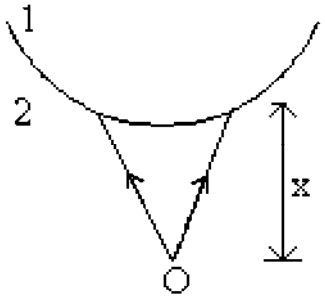
B) A – PR, B–QST, C–PRT, D –QS

C) A – QS, B–PR, C–PRT, D –QS

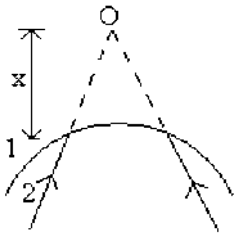
D) A – PR, B–QST, C–PRT, D –QST

19. In column I the surface refractivity and object are indicated and in column II possibilities about images are given. Match them for the R.I. relation $\mu_1 > \mu_2$.

COLUMN I	COLUMN II
A) 	P) Image will be real
B) 	Q) Image will be virtual

C) 

R) Image may be magnified

D) 

S) Image will be diminished.

T) Image may be real

A) A-PQRS, B-PQRS, C-PQRS, D-PQRST

B) A-PQRST, B-PQRST, C-PQRST, D-PQRST

C) A-PQRST, B-PQRS, C-PQRST, D-PQRST

D) A-PQRST, B-PQRST, C-PQRS, D-PQRS

20. Assuming the object for the optical entity given in column II may be either real or virtual match them will the type of image they can form given in column I .

Column I	**Column II**
(A)Real Image	(P) Converging lens
(B)Virtual Image	(Q) Diverging lens
(C)Magnified Image	(R) Concave mirror
(D)Diminished Image	(S) Convex mirror
	(T) Plane mirror

A) A – S, B – PS, C – RT, D – QR

B) A-PQRST, B-PQRST, C-PQRST, D-PQRST

C) A-P,R,Q,S B-P,R,Q,S, C-P,R,Q,S,T D- P,R,Q,S,T

D) A-P,R,Q,S,T B-P,R,Q,S,T C-P,R,S, D- P,R,Q,S

CHEMISTRY: **Max. Marks: 60**

SECTION – I

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

21. In a reaction $Cr_2O_7^{-2}$ was reduced to Cr^{+3} in acidic solution. Then which is / are true

A) The concentration of 0.1 M $Cr_2O_7^{-2}$ expressed in equivalent per litre is ----- 0.3

B) The concentration of 0.1 M $Cr_2O_7^{-2}$ expressed in equivalent per litre is ----- 0.6

C) Per mole of $Cr_2O_7^{-2}$ 6 moles of electrons are involved

D) oxidation number of Cr atom in $Cr_2O_7^{-2}$ is 6

22. Which is /are true

A) 3% $\left(W/v\right)$ H_2O_2 solution is approx 10'V'

B) In $C_6H_{12}O_6$ carbon is in zero oxidation state

C) 109% oleum means it contains 40% free SO_3

D) In Iodometric titration starch indicator is used

23. The sample(s) containing same no. of Na atom as there are Na atoms in 5.3 gm of Na_2CO_3, is/are

A) 4gm of NaOH B) 6.85 gm of NaCl

C) 0.25 mole of Na_2SO_4 D) 5.6 gm of Na_3PO_4

space for rough work

24. Excess of KI was added to 100 ml H_2O_2 solution of unknown strength alongwith sufficient H_2SO_4. Iodine liberated was titrated against 40 ml of 0.1 M hypo solution. The concentration of H_2O_2 solution is

A) 0.04 N B) 0.04 M C) 0.86 gm/L D) 0.03 M

25. Dichromate ion in acidic medium oxidizes stannous ion as:

$xSn^{2+} + yCr_2O_7^{2-} + zH^+ \rightarrow aSn^{4+} + bCr^{3+} + cH_2O$ then which is/are true

A) the value of $x:y$ is 1 : 3

B) the value of $x+y+z$ is 18

C) $a:b$ is 3 : 2

D) The value of $z-c$ is 7

26. Which of the following can act as oxidizing as well as reducing agent?

A) NH_3 B) HNO_3 C) H_2O_2 D) HNO_2

27. A 150 ml mixture of CO and CO_2 is passed through a tube containing excess of red hot charcoal. The volume become 200ml due to reaction $CO_2(g)+C(s) \rightarrow 2CO(g)$

A) Mole percent of CO_2 in the original mixture is 50

B) Mole fraction of CO in the original mixture is 0.66

C) The original mixture contains 60 ml of CO_2

D) The original mixture contain 50 ml of CO

28. Identify true statement (s)

A) The reaction $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$ is a disproportionation reaction.

B) In a disproportionation reaction neither oxidation nor reduction takes place.

C) Fluorine always exhibit an oxidation state of -1 in its compounds.

D) Oxidation numbers of 'S' in $Na_2S_2O_3$ is 2

SECTION - II

(COMPREHENSION TYPE)

This section contains **4 groups of questions.** Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.
Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

Paragraph for Questions 29 and 30:

Following titration method is given to determine total content of the species with variable oxidation states. Answer the question given at the end of it.

A quantity of 25.0 ml of solution containing both Fe^{2+} and Fe^{3+} ions is titrated with 25.0 mL of 0.0200 M $KMnO_4$ (in dilute H_2SO_4). As a result, all of the Fe^{2+} ions are oxidized to Fe^{3+} ions. Next 25mL of the original solution is treated with Zn metal. Finally, the solution requires 40.0 mL of the same $KMnO_4$ solution for oxidation to Fe^{3+}. $MnO_4^- + 5Fe^{2+} + 8H^+ \rightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$

29. Molar concentration of Fe^{2+} in the original solution is:

A) 0.01 M B) 0.02M C) 0.10M D) 0.20M

30. Molar concentration of Fe^{3+} in the original solution is

A) 0.06M B) 0.16M C) 0.032 M D) 0.012M

Paragraph for Questions 31 and 32:

A water is said to be a soft water if it produces sufficient foam with the soap and water that does not produce foam with soap is known as hard water.

Temporary hardness is due to presence of calcium and magnesium bicarbonate. It is simply removed by boiling as

$$Ca(HCO_3)_2 \xrightarrow{\Delta} CaCO_3 \downarrow + CO_2 \uparrow + H_2O$$

Temporary hardness can also be removed by addition of slaked lime, Ca $(OH)_2$

$$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 \downarrow + 2H_2O$$

Permanent hardness is due to presence of sulphate and chlorides of Ca, Mg etc. It is removed as $CaCl_2 + Na_2CO_3 \rightarrow CaCO_3 \downarrow + 2NaCl$

$$CaSO_4 + Na_2CO_3 \rightarrow CaCO_3 \downarrow + Na_2SO_4$$

100 ml of a sample of hard water after passing through cation exchange resin, required 20ml of 0.05M NaOH for neutralisation. One litre of same sample of water on treatment with sufficient lime gave 200mg of $CaCO_3$. Assume that the hardness is only due to Ca^{+2} ions.

31. The degree of permanent hardness in the given sample of water is

A) 100ppm B) 200ppm C) 300ppm D) 500ppm

32. The concentration of bicarbonate ions in ppm in the given sample of water is

A) 61ppm B) 122ppm C) 183ppm D) 100ppm

Paragraph for Questions 33 and 34:

The number of parts by weight of a substance that can combine with or displace 1.008 parts by weight of hydrogen or 35.5 parts by weight of chlorine or 8 parts of oxygen is known as the equivalent weight of a substance it is represented by E.

33. 3.0g of metal oxide converted to 5.0g of metal chloride. The equivalent weight of the metal is

A) 3.325 B) 66.50 C) 33.25 D) 25.33

34. One mole of chlorine combines with certain weight of metal giving 111gm of its chloride. The same amount of metal can displace 2gm hydrogen from an acid. The equivalent weight of metal is

A) 40 B) 20 C) 80 D) 10

Paragraph for Questions 35 and 36:

2 litre of 9.8 % (w/w) H_2SO_4 (d= 1.5 gm/ml) solution is mixed with 3 litre of 1 M KOH solution

35. The number of moles H_2SO_4 added are

A) 1 B) 2 C) 3 D) 0.5

36. The concentration of H^+ if solution is acidic or concentration of OH^- if solution is basic in the final solution is

A) 0 B) $\frac{3}{10}$ C) $\frac{3}{5}$ D) $\frac{2}{5}$

SECTION – III

(MATRIX MATCH TYPE)

This section contains **4 multiple choice questions.** Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

37. Given two mixtures:

I) $NaOH + Na_2CO_3$

II) $NaHCO_3 + Na_2CO_3$

100 ml of mixture I required 'W' and 'X' ml of 1 M HCl in separate titrations using phenolphthalein and Methyl orange indicators. While 100 ml of mixture II required 'Y' and 'Z' ml of same HCl solution in separate titration using same indicators.

Column I (Substance)		Column II (Molarity in solution)	
(A)	Na_2CO_3 in mixture I	(P)	$(2w-x)\times10^{-2}$
(B)	Na_2CO_3 in mixture II	(Q)	$(z-2y)\times10^{-2}$
(C)	$NaOH$ in mixture I	(R)	$y\times10^{-2}$
(D)	$NaHCO_3$ in mixture II	(S)	$(x-w)\times10^{-2}$

The correct answer is

A) A $\rightarrow$ S; B $\rightarrow$ R; C $\rightarrow$ P; D $\rightarrow$ Q

B) A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ Q; D $\rightarrow$ P

C) A $\rightarrow$ P; B $\rightarrow$ R; C $\rightarrow$ S; D $\rightarrow$ Q

D) A $\rightarrow$ S; B $\rightarrow$ P; C $\rightarrow$ R;D $\rightarrow$ Q

38. Match the Following

Column I		Column II	
(A)	100 ml of 0.2 M $AlCl_3$ solution +400 ml of 0.1 M HCl solution	(P)	Concentration of cation = 0.12 M
(B)	50 ml of 0.4 M KCl + 50 ml H_2O	(Q)	$\left[SO_4^{2-}\right] = 0.06M$
(C)	30 ml of 0.2 M K_2SO_4 + 70 ml H_2O	(R)	$\left[SO_4^{2-}\right] = 2.5M$
(D)	200 ml 24.5% (w/v) H_2SO_4	(S)	$\left[Cl^{-1}\right] = 0.2M$

The correct answer is

A) A-PS; B-SP; C-PQR; D-PR

B) A-PS; B-S; C-PQ; D-R

C) A-Q; B-S; C-PQ; D-PR

D) A -PQS; B -PR; C -SR; D –Q

39. Match the Following: Identify equivalent weight of underlined species from redox reactions

Column I	Column II
(A) $\underline{P_2H_4} \rightarrow PH_3 + P_4H_2$	(P) $E = \frac{3M}{4}$
(B) $\underline{I_2} \rightarrow I^- + IO_3^-$	(Q) $E = \frac{3M}{5}$
(C) $MnO_4^- + Mn^{2+} + H_2O \rightarrow \underline{Mn_3O_4} + H^+$	(R) $E = \frac{15M}{26}$
(D) $\underline{H_3PO_2} \rightarrow PH_3 + H_3PO_3$	(S) $E = \frac{5M}{6}$

The correct answer is

A) A $\rightarrow$ S; B $\rightarrow$ Q; C $\rightarrow$ P; D $\rightarrow$ R

B) A $\rightarrow$ R; B $\rightarrow$ Q; C $\rightarrow$ R; D $\rightarrow$ P

C) A $\rightarrow$ P; B $\rightarrow$ R; C $\rightarrow$ S; D $\rightarrow$ Q

D) A $\rightarrow$ S; B $\rightarrow$ Q; C $\rightarrow$ R; D $\rightarrow$ P

40. List –I List-II (at STP)

A) 10gm $CaCO_3 \xrightarrow[decomposition]{\Delta}$ P) 6×10^{22} CO_2 molecules

B) $10.6gm\ Na_2CO_3 \xrightarrow{excess\ HCl}$ Q) 1.2×10^{23} CO_2 molecules

C) 2.4 gm C $\xrightarrow[combustion]{excess\ O_2}$ R) 4.48 lit CO_2

D) 5.6 gm CO $\xrightarrow[combustion]{excess\ O_2}$ S) 2.24 lit CO_2

T) 0.448 lit CO_2

The correct answer is

A) A-SPQ; B-SPR; C-RQ; D-RQS

B) A-SP; B-SP; C-RQ; D-RQ

C) A-SPQ; B-SPQ; C-SRQ; D-PRQ

D) A-SPQ; B-SPR; C-RSQ; D-RQ

MATHEMATICS: **Max. Marks: 60**

SECTION – I

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

41. $S_n = 1^2+2^2x+3^2x^2+4^2x^3+.....+\infty$, then $S_\infty=$

A) 12 for x=1/2 B) 9/2 for x=1/3

C) 16 for x=1/2 D) 64/9 for x=1/4

42. If $x^2+y^2+z^2=1$, then the value of x+2y+3z cannot be greater than

A) 3 B) 4 C) 5 D) 2

43. The series $\frac{8}{5}+\frac{16}{65}+\frac{24}{325}+.....+\frac{8n}{4n^4+1}$

A) The sum to infinite number of terms of the series is 2

B) The sum to n terms of the series is less than $2 \forall n \in N$

C) The sum to n terms of the series cannot be an integer for any $n \in N$

D) The sum to infinite number of terms of the series is 13/3

44. If $ax^2 + \frac{b}{x} \le c \forall x > 0, a > 0 \, and \, b > 0,$ then $27ab^2$ is

A) less than or equal to 4 for $c \le 1$

B) less than or equal to 32 for $c \le 2$

C) greater than or equal to 108 for $c \le 3$

D) greater than or equal to 64 for $c \le 2$

45. If the arithmetic mean of two positive numbers a&b (a>b) is twice their geometric mean, then a:b can be

A) $2+\sqrt{3}:2-\sqrt{3}$ B) $7+4\sqrt{3}:1$ C) $1:7-4\sqrt{3}$ D) $2:\sqrt{3}$

46. If both the roots of the equation x^2-6ax+2-2a+$9a^2$=0 exceeds 3 then

A) a cannot be less than1 B) a>$\frac{11}{9}$

C) a>$\frac{3}{2}$ D) a<$\frac{5}{2}$

47. If a,b,c are in H.P then the expression $E = \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ equals

A) $\frac{2}{bc}-\frac{1}{b^2}$ B) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$ C) $\frac{3}{b^2}-\frac{2}{ab}$ D) $\frac{3}{b^2}+\frac{2}{ab}$

48. If (1+3+5+.....+p) + (1+3+5+.....+q) = (1+3+5+.....+r) where each set of paranthesis Contain the sum of consecutive odd integers and p>6 then

A) The smallest possible value of p+q+r =21

B) The maximum value of p+q+r=21

C) p+q+r can attain the value 45

D) p+q+r is an odd prime integer

SECTION - II
(COMPREHENSION TYPE)

This section contains **4 groups of questions.** Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.
Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

Paragraph for Questions 49 and 50:

The sum of three terms of a strictly increasing G.P. is αs and sum of the squares of these terms is S^2

49. α^2 lies

A) (1/3, 2) B) (1,2) C) (1/3,3) D) $\left(\frac{1}{3},1\right)\cup(1,3)$

50. If α =1/2, S=20, then the greatest value of the first term is

A) 10/3 B) 7/3 C) 1/3 D) 3

Paragraph for Questions 51 and 52:

Let ABCD is a unit square and $0 < \alpha < 1$. Each side of the square is divided in the ratio $\alpha : 1-\alpha$ .These points are connected to obtain another square. The sides of new square are divided in the ratio $\alpha : 1-\alpha$ and points are joined to obtain another square. The process is continued indefinitely. Let a_n denote the length of side and A_n be the nth square. (Including given Square)

51. If α=1/3, the least value of n for which A_n<1/10 is

A) 4 B) 5 C) 6 D) 7

52. The value of α for which side of nth square equals the diagonal of (n+1) th square is

A) 1/3 B) 1/4 C) 1/2 D) $1/\sqrt{2}$

Paragraph for Questions 53 and 54:

We know that, if $a_1, a_2, ..., a_n$ are in H.P., then $\frac{1}{a_1}, \frac{1}{a_2}, ..., \frac{1}{a_n}$, are in A.P. and vice-versa. If $a_1, a_2, ..., a_n$ are in A.P. with common difference d, then for any b(>0), the number $b^{a_1}, b^{a_2}, b^{a_3}, ..., b^{a_n}$ are in G.P. with common ratio b^d. If $a_1, a_2, ..., a_n$ are positive and in G.P. with common ratio r, then for any base b(b>0), $\log_b a_1$, $\log_b a_2$, ..., $\log_b a_n$ are in A.P. with common difference $\log_b r$.

53. If x,y,z are respectively the p^{th},q^{th} and r^{th} terms of an A.P., as well as a G.P., then the value of $x^{y-z}.y^{z-x}.z^{x-y}$ is

A) 1 B) -1 C) 0 D) 2

54. If a,b,c are in H.P., then $4^{-a^{-1}},4^{-b^{-1}},4^{-c^{-1}}$ are in

A) A.P. B) G.P. C) H.P. D) none of these

Paragraph for Questions 55 and 56:

Let A_1,G_1,H_1 denote the arithmetic, geometric and harmonic means, respectively of two distinct positive numbers. For $n\geq 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

55. Which of the following statements is a correct statement?

A) $G_1 > G_2 > G_3 > .....$ B) $G_1 < G_2 < G_3 < .....$

C) $G_1 = G_2 = G_3 = ...$ D) $G_1 < G_2 < G_3 < .....and\ G_1 > G_2 > G_3 > .....$

56. Which of the following statements is a correct statement?

A) $H_1 > H_2 > H_3 > .....$

B) $H_1 < H_2 < H_3 < ....$

C) $H_1 > H_3 > H_5 > .....and\ H_2 < H_4 < H_6 < ....$

D) $H_1 < H_3 < H_5 < .....and\ H_2 > H_4 > H_6 > ....$

SECTION – III

(MATRIX MATCH TYPE)

This section contains **4 multiple choice questions.** Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

57.

Column – 1	Column – 2
(A) If a,b,c are positive real numbers such that $49\left(4a^2+9b^2+c^2\right)=36\left(a+b+c\right)$ then $36\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)=$	P) 1210
(B) Let $\{a_1,a_2,....\}$ be a sequence such that $a_1=1$ and $a_n-a_{n-1}=n^2\forall n\geq 2$ then $\sum_{i=1}^{10}a_i$ is equal to	Q) 270
(C) If $\sum_{k=1}^{15}\left(\frac{1}{2k}-\frac{1}{k+1}+\frac{1}{2\left(k+2\right)}\right)=a$ then 1088 a=	R) 229
(D) If $S_n=\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\frac{9}{400}+....$ to n term then $\frac{1}{1-S_{40}}$ is equal to	S) 1681

A) $A\rightarrow R;B\rightarrow P;C\rightarrow S;D\rightarrow Q$ B) $A\rightarrow R;B\rightarrow P;C\rightarrow Q;D\rightarrow S$

C) $A\rightarrow R;B\rightarrow S;C\rightarrow Q;D\rightarrow P$ D) $A\rightarrow R;B\rightarrow Q;C\rightarrow P;D\rightarrow S$

space for rough work

58. Column – 1 | Column – 2

(A) If a,b,c be positive numbers then $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ | P) 4

Must be greater than or equal to.

(B) If H be the H.M. and g be the G.M. of two positive | Q) 9

Numbers a and b such that h:g=4:5, then $\frac{a}{b}$ can be equal to

(C) If $S=\sum_{r=0}^{\infty}\frac{1}{2^r}$ and $S_{n+1}=\sum_{r=0}^{n}\frac{1}{2^r}$ and $S-S_{n+1}<10^{-3}$ | R) 10

Then n can be

(D) If $(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right).....\left(1+x^{128}\right)=\sum_{r=0}^{n}x^r$ | S) 255

Then n is equal to

A) A $\rightarrow$ P,Q;B $\rightarrow$ P;C $\rightarrow$ R,S;D $\rightarrow$ S

B) A $\rightarrow$ P,Q;B $\rightarrow$ S;C $\rightarrow$ R,S;D $\rightarrow$ P

C) A $\rightarrow$ P,Q;B $\rightarrow$ P;C $\rightarrow$ S;D $\rightarrow$ S

D) A $\rightarrow$ P;B $\rightarrow$ P;C $\rightarrow$ R,S;D $\rightarrow$ S

59. Column – 1 | Column – 2

(A) If a,b,c are positive real numbers then the least value of $\frac{a^3}{4b}+\frac{b}{8c^2}+\frac{1+c}{2a}$ is $\frac{x}{y}(x,y\ coprime\ natural\ numbers)$ where $x+y=$ — P) 2

(B) If a,b,c are positive real numbers then the least value of $\frac{(a+b)(b+c)(c+a)}{abc}$ is — Q) 5

(C) If a,b,c are positive real numbers such that a+b+c=1 then the least value of $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is — R) 8

(D) If a,b,c are the sides of triangle then $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}<k$ where least positive integral value of k is — S) 9

A) A $\rightarrow$ S; B $\rightarrow$ R; C $\rightarrow$ Q; D $\rightarrow$ P

B) A $\rightarrow$ S; B $\rightarrow$ Q; C $\rightarrow$ R; D $\rightarrow$ P

C) A $\rightarrow$ S; B $\rightarrow$ R; C $\rightarrow$ R; D $\rightarrow$ P

D) A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ R; D $\rightarrow$ P

60. Column – 1 Column – 2

(A) If three unequal numbers a,b,c are in A.P. and b-a,c-b,a Are in G.P. then $\frac{a^3+b^3+c^3}{3abc}$ is equal to — P) 1

(B) let x be the arithmetic mean and y,z be two geometric Means between any two positive numbers, then $\frac{y^3+z^3}{2xyz}$ Is equal to — Q) 4

(C) If a,b,c be three positive number which form three Successive terms of a G.P. and c>4b-3a, then the common ratio of the G.P. can be equal to. — R) 2

(D) number of integral values of x satisfying inequality $-7x^2+8x-9>0$ is — S) 0

A) A $\rightarrow$ R; B $\rightarrow$ Q; C $\rightarrow$ P; D $\rightarrow$ S

B) A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ Q; D $\rightarrow$ P

C) A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ R; D $\rightarrow$ P

D) A $\rightarrow$ R; B $\rightarrow$ P; C $\rightarrow$ Q; D $\rightarrow$ S

Master JEE CLASSES

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KEY SHEET

PHYSICS

1	**ABD**	2	**BC**	3	**C**	4	**ABC**	5	**ABCD**
6	**BCD**	7	**AC**	8	**BCD**	9	**B**	10	**C**
11	**C**	12	**D**	13	**B**	14	**C**	15	**A**
16	**B**	17	**C**	18	**B**	19	**D**	20	**D**

CHEMISTRY

21	**BCD**	22	**ABCD**	23	**A**	24	**A**	25	**BCD**
26	**CD**	27	**B**	28	**ACD**	29	**C**	30	**A**
31	**C**	32	**B**	33	**B**	34	**B**	35	**C**
36	**C**	37	**A**	38	**B**	39	**D**	40	**B**

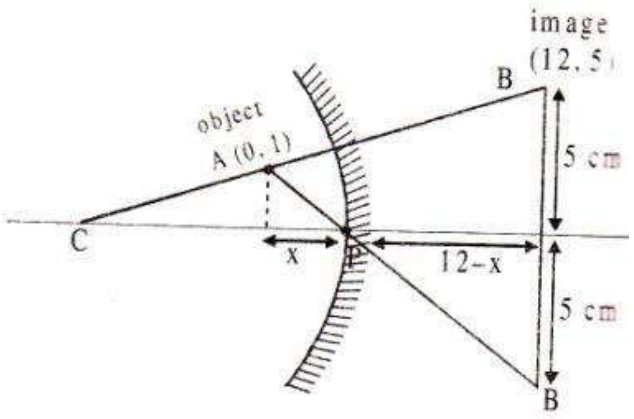
MATHS

41	**AB**	42	**BC**	43	**ABC**	44	**AB**	45	**ABC**
46	**AB**	47	**ABC**	48	**AC**	49	**D**	50	**A**
51	**B**	52	**C**	53	**A**	54	**B**	55	**C**
56	**B**	57	**B**	58	**A**	59	**C**	60	**D**

SOLUTIONS

PHYSICS

1. It is negative when it lies left of the main scale.
2. Conceptual
3. Conceptual
4. Conceptual
5. Join, object and image with a line that intersects the principle axis at point c. (centre of curvature) From the similar triangles



$\frac{1}{x} = \frac{5}{12-x} \Rightarrow x = 2\,cm$

Apply mirror formula we get $f = 2.5\,cm$

6. from given condition m $\mu_1 > \mu_2, \mu_1 > \mu_3$ $\sin ce,\ \mu_1, \mu_2\ and\ \mu_3 > 1$

Hence, $\mu_1^2 > \mu_2^2,\ \mu_1^2 > \mu_3^2.$

7. Conceptual
8. Apply Snells law : $\mu_2 \sin i - \mu_1 \sin r \Rightarrow \sin i - k \sin r$

From the given graph, angle of deviation decreases and becomes zero at $k = k_2$

Hence, $\theta_1 = |r-i| = \frac{\pi}{6}$(By geometry)

$\Rightarrow$ at $k = k_2, \theta = |r-i| = 0$

$\Rightarrow$ when $k = \infty$, r = 0, by the Snells law. $\theta_2 |r-1| = 1 = \frac{\pi}{3}$

$\Rightarrow k_1 =$must be less than $k_2$from the given graph

9. $Least\ Count = \frac{Pitch}{No.\ of\ divisions\ on\ Circular\ scale}$

$$Pitch = \frac{Linear\ dis\tan ce\ moved}{No.\ of\ rotations}$$

10. Conceptual

11. Using $\frac{1}{v} - \frac{1}{[2f + f\cos\omega t]} = \frac{-1}{f}$

We get $v = -\left(\frac{2+\cos\omega t}{1+\cos\omega t}\right).f$

12. The ball coincides with its image at the centre of curvature

13. LC $= \frac{1}{50} = 0.02$ mm

14. The instrument has positive zero error

e = + 3 × 0.02 = 0.06 mm

15. Conceptual

16. Conceptual

17. Conceptual

18. Conceptual

19. Conceptual

20. Conceptual

CHEMISTRY

24. 40 mL of 0.1 M hypo $\Rightarrow 4\times10^{-3}$ equivalent of $H_2O_2 \Rightarrow 4\times17\times10^{-3}$ g in 100 mL $\Rightarrow$ 0.68 g per L (or) 0.02 M (or) 0.04 N

25. $3Sn^{2+} + 14H^+ + Cr_2O_7^{2-} \rightarrow 3Sn^{3+} + 2Cr^{3+} + 7H_2O$

26.

29. Only Fe^{2+} is oxidized and MnO_4^- is reduced to Mn^{2+}

$25\times M\left(Fe^{2+}\right) = 25\times0.02\times5\left(MnO_4^-\right)$

M = 0.10M

30. Total Fe^{2+} in second part (including that of from Fe^{3+})

$25\times y = 40\times0.02\times5$

y = 0.16 M = 0.16 N

Fe^{3+} = 0.16 – 0.10(Fe^{2+})

= 0.06 M

31, 32.

From the solution, $Ca(HCO_3)_2 + CaO \longrightarrow 2CaCO_3 + H_2O$

200mg of $CaCO_3$ is given by one m mole of $Ca(HCO_3)_2$ per Lt.

$\therefore$ Degree of temporary hardness = 100 ppm

When water is passed through cation exchange resin all the cations are exchanged with 'H^+' ions and HCO_3^- ions remain same.

milli eq of HCO_3^- + milli eq. of Ca^{+2} = milli eq. of NaOH

milli eq. of Ca^{+2} = 1-0.2 = 0.8

$\therefore$ milli moles of Ca^{+2} = 0.4 per 100 ml

Total degree of hardness = 400 ppm

Hence degree of permanent hardness = 300 ppm

33, 34

MW of MCl_2 = 111g

M + 71 = 111 M = 111 – 71 = 40

2gm of H_2 displaced by 40gm of metal

1gm of H_2 displaced by?

37. <u>Mixture I:</u>

End point with phenolphthalein (disappearance of pink colour) corresponds to the neutralisation of $NaOH$ and half- neutralisation of Na_2CO_3.

$NaOH + HCl \rightarrow NaCl + H_2O$ $Na_2CO_3 + HCl \rightarrow NaHCO_3 + NaCl$

End point with Methyl orange (Appearance of Red colour) corresponds to the neutralisation of $NaOH$ and Na_2CO_3.

$NaOH + HCl \rightarrow NaCl + H_2O$

$Na_2CO_3 + 2HCl \rightarrow 2NaCl + CO_2 + H_2O$

Volume of HCl required for neutralisation of $Na_2CO_3 = 2(x-w)$

Normality of $Na_2CO_3 = \dfrac{1\times 2(x-w)}{100}$

$2(x-w)\times 10^{-2}$ Molarity of $Na_2CO_3 = (x-w)\times 10^{-2}$

Volume of HCl req. for neutralisation of $NaOH$

$= w-(x-w) = (2w-x)ml$

Hence, molarity of $NaOH = \frac{(2w-x)\times 1}{100}$

$= (2w-x)\times 10^{-2} M$

Mixture II:

End point with phenolphthalein corresponds to half- neutralisation of Na_2CO_3 as

$Na_2CO_3 + HCl \rightarrow NaHCO_3 + NaCl$

Volume of HCl req. for complete neutralisation of Na_2CO_3 = '2y' ml

$\therefore Molarity\ of\ Na_2CO_3 = \frac{1}{2}\times\frac{2y}{100} = y\times 10^{-2}$

End point with Methyl orange corresponds to neutralisation of $NaHCO_3$

Hence, volume required for neutralisation of $NaHCO_3$ present initially = $(z-2y)ml$

$\therefore$ Molarity of $NaHCO_3 = \frac{(z-2y)}{100} = (z-2y)\times 10^{-2}$

40. A) $CaCO_3 \rightarrow CaO + CO_2$ B) $Na_2CO_3 + 2HCl \rightarrow 2NaCl + H_2O + CO_2$

C) $C + O_2 \rightarrow CO_2$ D) $CO + \frac{1}{2}O_2 \rightarrow CO_2$

MATHS

41. $S_n(1-x) = 1^2 + (2^2-1^2)x + (3^2-2^2)x^2 + (4^2-3^2)x^3 + ...$

$= 1 + 3x + 5x^2 + 7x^2 + ... \ in(A.G.P)$

$S_\infty = \frac{1+x}{(1-x)^3}$

42. $(x+2y+3z)^2 \le (1^2+2^2+3^2)(x^2+y^2+z^2)$

Cauchy-schwarz inequality

$x+2y+3z \le \sqrt{14}$

43. $T_r = \frac{8r}{4r^4+1} = \frac{8r}{(2r^2+2r+1)(2r^2-2r+1)}$

$= 2\left\{\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2+2r+1)(2r^2-2r+1)}\right\}$

$= 2\left\{\frac{1}{2r^2-2r+1} - \frac{1}{2r^2+2r+1}\right\} = 2\left\{\frac{1}{v_r} - \frac{1}{v_{r+1}}\right\}$

$$\sum_{r=1}^{\infty} T_r = 2\left[\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{13}\right)+....\right] = 2$$

$$1 < \sum_{r=1}^{n} T_r < 2$$

44. $$\frac{ax^2+\frac{b}{2x}+\frac{b}{2x}}{3} \geq \left(ax^2.\frac{b}{2x}.\frac{b}{2x}\right)^{1/3}$$

$$\frac{1}{3}\left(ax^2+\frac{b}{x}\right) \geq \left(\frac{ab^2}{4}\right)^{1/3}$$

$$3\left(\frac{ab^2}{4}\right)^{1/3} \leq c$$

AM $\geq$ GM

$$3\left(\frac{ab^2}{4}\right)^{1/3} \leq ax^2+\frac{b}{x} \leq c$$

$$27\frac{\left(ab^2\right)}{4} \leq c^3$$

$$27ab^2 \leq 4c^3$$

45. $$\frac{a+b}{2\sqrt{ab}} = 2 \Rightarrow \sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}} = 4$$

$$\frac{a}{b} = 7 \pm 4\sqrt{3}$$

$$a > b$$

$$\therefore \frac{a}{b} = 7+4\sqrt{3}$$

46. $$\left(x-3a\right)^2 = 2a-2$$

$$x = 3a \pm \sqrt{2a-2}$$

$$3a-\sqrt{2a-2} > 3$$

$$3\left(a-1\right) > \sqrt{2}\sqrt{a-1}$$

$$\therefore a > 1, \sqrt{a-1} > \frac{\sqrt{2}}{3}$$

47. $$E = \left(\frac{1}{c}+d\right)\left(\frac{1}{c}-d\right) = \frac{1}{c^2}-d^2$$

$$= \frac{1}{c^2}-\left(\frac{1}{c}-\frac{1}{b}\right)^2 = \frac{2}{bc}-\frac{1}{b^2}$$

Similarly $E=\frac{1}{c^2}-\frac{1}{4}\left(\frac{1}{c}-\frac{1}{a}\right)^2=\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$

$E=\left(\frac{2}{b}-\frac{1}{a}\right)^2-\left(\frac{1}{b}-\frac{1}{a}\right)^2=\frac{3}{b^2}-\frac{2}{ab}$

48. $1+3+5+.....+(2k-1)=k^2$

$\therefore\left(\frac{p+1}{2}\right)^2+\left(\frac{q+1}{2}\right)^2=\left(\frac{r+1}{2}\right)^2$

$\therefore(p+1)^2+(q+1)^2=(r+1)^2 \quad p+1>7$

$(p+1,q+1,r+1)\Rightarrow$ pythagorean triplet.

The first pythagoream triplet contains a number > 7 is (6,8,10)

P=7 q=5 r=9

P+q+r=21

Also when p+1=16, q+1=12, r+1=20

P+q+r= 15+11+19=45

49. $a\left(\frac{1}{r}+1+r\right)=\alpha s \qquad (1)$

$a^2\left(\frac{1}{r^2}+1+r^2\right)=s^2 \qquad (2)$

Divide (2) and (1) to obtain

$a\left(\frac{1}{r}+1+r\right)=\frac{s}{\alpha}+2a \qquad (3)$

From (2) and (3)

$2a=s\left(\alpha-\frac{1}{\alpha}\right)=s\left(\frac{\alpha^2-1}{\alpha}\right)$

Putting this in (2) we get

$\frac{(\alpha^2-1)^2}{4\alpha^2}\left(\frac{1}{r^2}+1+r^2\right)=1$

$\Rightarrow\left(r-\frac{1}{r}\right)^2+3=\frac{4\alpha^2}{(\alpha^2-1)^2} \qquad (4)$

$\Leftrightarrow 3\alpha^4-10\alpha^2+3<0$

$\Leftrightarrow(3\alpha^2-1)(\alpha^2-3)<0$

$\Leftrightarrow 1/3 < \alpha^2 < 3$

But $\alpha^2 = 1 \Rightarrow a = 0$. Not possible.

50. $a\left(\frac{1}{r}+1+r\right)=\left(\frac{1}{2}\right)(20)$

$\Rightarrow \frac{10}{a}=\left(\sqrt{r}-\frac{1}{\sqrt{r}}\right)^2+3 \geq 3$

$\Rightarrow a \leq 10/3$

Also, $a^2\left(\frac{1}{r^2}+1+r^2\right)=20^2$

$\Rightarrow a \leq 20/\sqrt{3}$

Thus, $\Rightarrow a \leq 10/3$

51. $a_1 = 1 and\, a^2{}_{n+1} = \left[(1-\alpha)^2+\alpha^2\right] a^2{}_n$

$=\frac{5}{9} a_n{}^2 = \left(\frac{5}{9}\right)^n$

Now, $A_n = a_n{}^2 < \frac{1}{10}$

$\Rightarrow \left(\frac{9}{5}\right)^{n-1} > 10$

$\Rightarrow 9^{n-1} > 2(5^n)$

$\Rightarrow n \geq 5$

52. Diagonal of (n+1)th square = $\sqrt{2} a_{n+1}$

Now $a_n = \sqrt{2} a_{n+1}$

$\Rightarrow a_n{}^2 = 2 a_{n+1}{}^2$

$\Rightarrow a_n{}^2 = 2\left(2\alpha^2 - 2\alpha + 1\right) a_n{}^2$

$\Rightarrow \alpha = 1/2$

53. Let the base be taken as e. Since x,y,z are terms of a G.P. (say with common ratio t), lnx, lny,lnz are in À.P. with common difference lnt. Also, x,y,z terms of A.P. (say with common difference d)

Hence x-y=(p-q)d etc. And lnx-lny= (p-q)lnt. Etc.

Let $E = x^{y-z}.y^{z-x}.z^{x-y}$ so that

$\ln E = (y-z)\ln x + (z-x)\ln y + (x-y)\ln z$

$= (q-r)d\ln x + (r-p)d\ln y + (p-q)d\ln z$

$= d\left[p\left(\ln z - \ln y\right) + q\left(\ln x - \ln z\right) + r\left(\ln y - \ln x\right)\right]$

$= d \ln t\left[p\left(r-q\right) + q\left(p-r\right) + r\left(q-p\right)\right] = 0 \Rightarrow E = 1$

54. (B) Here a,b,c are in H.P.

$\Rightarrow a^{-1}, b^{-1}, c^{-1}$ are in A.P. $\Rightarrow \left(\frac{1}{4}\right)^{a^{-1}}, \left(\frac{1}{4}\right)^{b^{-1}}, \left(\frac{1}{4}\right)^{c^{-1}}$ are in G.P.

$\Rightarrow 4^{-a^{-1}}, 4^{-b^{-1}}, 4^{-c^{-1}}$ are in G.P.

55. Let two distinct positive numbers be a and b.

$A_1 = \frac{1}{2}\left(a+b\right), G_1 = \sqrt{ab}, H_1 = \frac{2ab}{a+b}$

$A_n = \frac{1}{2}\left(A_{n-1} + H_{n-1}\right), G_n = \sqrt{A_{n-1} + H_{n-1}}$

$H_n = \frac{2A_{n-1} + H_{n-1}}{A_{n-1} + H_{n-1}}$

$for \;\; n \geq 2$

$A_n H_n = A_{n-1} + H_{n-1}$

$thus, A_1 H_1 = A_2 H_2 = A_3 + H_3 = ...$

$\Rightarrow G_1^2 = G_2^2 = G_3^2 = ...$

$G_1 = G_2 = G_3 = ...$

56. As a and b are distinct, $A_1 > H_1$

$\Rightarrow A_1 > A_2 > H_2 > H_1$

$\Rightarrow A_1 > A_2 > A_3 > H_3 > H_2 > H_1$

And so on.

Thus, $A_1 > A_2 > A_3 > .....$

And $H_1 < H_2 < H_3 < .....$

57. A-q;B-p;C-q;D-s

(A) $\frac{\left[\left(s-a\right)^2 + \left(s-a\right) + 1\right]}{\left(s-a\right)} \frac{\left[\left(s-b\right)^2 + 2\left(s-b\right) + 1\right]}{\left(s-b\right)} \frac{\left[\left(s-c\right)^2 + 3\left(s-c\right) + 1\right]}{\left(s-c\right)} =$

$\left[\left(s-a\right) + \frac{1}{s-a} + 1\right]\left[\left(s-b\right) + \frac{1}{s-b} + 2\right]\left[\left(s-c\right) + \frac{1}{s-c} + 3\right]$

(B) $a_i = 1^2 + 2^2 + ...i^2 = \frac{i\left(i+1\right)\left(2i+1\right)}{6}$

$S = \sum_{i=1}^{10} a_i = \sum_{i=1}^{10} \frac{i\left(i+1\right)\left(2i+1\right)}{6} = 1210$

(C) $T_n = \dfrac{n^2}{500+3n^3}$

Let $U_n = \dfrac{1}{T_n} = \dfrac{500}{n^2} + 3n$

$$\frac{dT_n}{dn} = \frac{\left(500+3n^3\right)2n - n^2 9n^2}{\left(500+3n^3\right)^2} = \frac{n\left(1000-3n^3\right)}{\left(500+3n^3\right)^2} = 0$$

$$\Rightarrow n = \left(\frac{1000}{3}\right)^{1/3}$$

Now, $6 < \left(\dfrac{1000}{3}\right)^{1/3} < 7$

(D) $T_r = \dfrac{2r+1}{r^2\left(r+1\right)^2} = \dfrac{\left(r+1\right)^2 - r^2}{r^2\left(r+1\right)^2} = \dfrac{1}{r^2} - \dfrac{1}{\left(r+1\right)^2}$

$$S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} \left[V_r - V_{r+1}\right]$$

$$= V_1 - V_{n+1} = 1 - \frac{1}{\left(n+1\right)^2}$$

58. A-p,q;B-p;C-r,s;D-s

(A) $A.M \geq H.M \Rightarrow \dfrac{a+b+c}{3} \geq \dfrac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$$\Rightarrow \left(a+b+c\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$$

(B) $h = \dfrac{2ab}{a+b}, g = \sqrt{ab} \Rightarrow \dfrac{h}{g} = \dfrac{2ab}{\left(a+b\right)\sqrt{ab}}$

$$\Rightarrow \frac{4}{5} = \frac{2\sqrt{ab}}{a+b}$$

$$\Rightarrow \frac{9}{1} = \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} \Rightarrow \left|\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right| = 3$$

$$or \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \pm 3$$

$$or\ a:b = 1:4\ or\ 4:1$$

(C) $S = \dfrac{1}{1-\frac{1}{2}} = 2\ and\ S_{n+1} = \dfrac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}} = 2 - \dfrac{1}{2^n}$

$\therefore S - S_{n+1} = \frac{1}{2^n} < \frac{1}{1000} \Rightarrow 2^n > 1000$

$But\ 2^9 < 1000 < 2^{10}$

$\therefore n \geq 10$

(D) $(1+x)(1+x^2)(1+x^4)(1+x^8).....(1+x^{128})$

$\frac{1-x^{256}}{1-x} = \sum_{r=0}^{255} x^r \Rightarrow n = 255$

59. A-r; B-q; C-q; D-p

(A) $A_p = a + (p-1)d$ .....(1)

$A_q = a + (q-1)d$ .....(2)

$A_r = a + (r-1)d$ .....(3)

$A_s = a + (s-1)d$ .....(4)

$A_q = kA_p$

$A_r = k^2 A_p \qquad (A_p, A_q, A_r, A_s \ in\ G.P.)$

$A_s = k^3 A_p$

$(p-q) = \frac{A_p - A_q}{d} = A_p \frac{(1-k)}{d}\ from(1)\ and\ (2)$

$(q-r) = A_p k \frac{(1-k)}{d}\ from\ (2)\ and\ (3)$

$(r-s) = A_p k^2 \frac{(1-k)}{d}$

$\Rightarrow p-q, q-r, r-s\ are\ in\ G.P.$

(B) In x, In y, In z are in G.P.

$\Rightarrow$ In(In x), In(In y), In(In z) are in A.P.

$\Rightarrow$ 2x+ In(In x),3x+ In(In y),4x+ In(In z) are in A.P.

(C) $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = ... = k(say)$

$\frac{1}{x}\log a = \frac{1}{y}\log b = \frac{1}{z}\log c = ... = \log k$

$But \log a, \log b, \log c \Rightarrow A.P$

$x\log k, y\log k, z\log k \Rightarrow A.P$

$x, y, z \Rightarrow A.P$

(D) $\frac{(b-c)^2 + (a-b)^2 + (c-a)^2}{3}$

$$\Rightarrow \frac{(b+c-2a)^2+(c+a-2b)^2+(a+b-2c)^2}{3}$$

$$\Rightarrow (b+c-2a)^2-(b-c)^2+(c+a-2b)^2-(c-a)^2+(a+b-2c)^2-(a-b)^2=0$$

$$a=b=c$$

60. (A) a,b,c => b-d, b, b+d

b-a=b-(b-d) = d

c-a = b+d-b = d

a = b-d

d,d,b-d => G.P.

b=2d

$$\frac{a^3+b^3+c^3}{3abc}= =\frac{36d^3}{18d^3}=2$$

(B) $x=\frac{a+b}{2}, y=a\left(\frac{b}{a}\right)^{1/3}, z=a\left(\frac{b}{a}\right)^{2/3}$

$y^3+z^3=a^2b+ab^2$ $\qquad xyz=\frac{a+b}{2}a^{2/3}b^{1/3}a^{1/3}b^{2/3}$

$=ab(a+b)$ $\qquad =\frac{(a+b)ab}{2}$

$$\frac{y^3+z^3}{2xyz}=\frac{ab(a+b)}{2\left(\frac{a+b}{2}\right)ab}=1$$

(C) a,ar,ar^2, ar^2>4ar-3a (a>0)

r^2-4r+3>0, r^2>4r-3

r<1 or r>3

(D) 7x^2-8x+9<0 No real solutions