(aa) If θ is the angle between the line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ and the plane $Ax+By+Cz+D=0$, then

$$\sin\theta=\frac{|aA+bB+cC|}{\sqrt{a^2+b^2+c^2}\sqrt{A^2+B^2+C^2}}$$

(ab) Length of perpendicular from (x_1, y_1, z_1) to the plane $ax+by+cz+d=0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

(ac) The equation of a sphere with center at the origin and radius 'a' is $|\vec{r}|=a$ or $x^2+y^2+z^2=a^2$

(ad) Equation of a sphere with center (α,β,γ) and radius 'a' is $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2=a^2$

(ae) Vector equation of the sphere with center $\vec{c}$ and radius 'a' is $|\vec{r}-\vec{c}|=a$ or $(\vec{r}-\vec{c})\cdot(\vec{r}-\vec{c})=a^2$

(af) General equation of sphere is $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ whose center is $(-u,-v,-w)$ and radius is $\sqrt{u^2+v^2+w^2-d}$

(ag) Equation of a sphere concentric with $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ is $x^2+y^2+z^2+2ux+2vy+2wz+\lambda=0$, where λ is a real number.

Solved Examples

JEE Main/Boards

Example 1: Find the coordinates of the point which divides the join of P(2, –1, 4) and Q(4, 3, 2) in the ratio 2 : 3 (i) internally (ii) externally

Sol: By using section formula we can obtain the result.

Let R(x, y, z) be the required point

(i) $x=\frac{2\times4+3\times2}{2+3}=\frac{14}{5}$; $y=\frac{2\times3+3\times(-1)}{2+3}=\frac{3}{5}$

$z=\frac{2\times2+3\times4}{2+3}=\frac{16}{5}$

So, the required point is $R\left(\frac{14}{5},\frac{3}{5},\frac{16}{5}\right)$

(ii) $x=\frac{2\times4-3\times2}{2-3}=-2$; $y=\frac{2\times3-3\times(-1)}{2-3}=-9$

$z=\frac{2\times2-3\times4}{2-3}=8$

Therefore, the required point is R(–2. –9, 8)

Example 2: Find the points on X-axis which are at a distance of $2\sqrt{6}$ units from the point (1, –2, 3)

Sol: Consider required point is P(x, 0, 0), therefore by using distance formula we can obtain the result.

Let P(x, 0, 0) be a point on X-axis such that distance of P from the point (1, –2, 3) is $2\sqrt{6}$

$\Rightarrow\sqrt{(1-x)^2+(-2-0)^2+(3-0)^2}=2\sqrt{6}$

$\Rightarrow(x-1)^2+4+9=24$ $\quad\Rightarrow(x-1)^2=11$

$\Rightarrow x-1=\pm\sqrt{11}$ $\quad\Rightarrow x=1\pm\sqrt{11}$

Example 3: If a line makes angles α,β,γ with OX, OY, OZ, respectively, prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

Sol: Same as illustration 2.

Let ℓ,m,n be the d.c.'s of the given line, then

$\ell=\cos\alpha, m=\cos\beta, n=\cos\gamma$

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow(1-\sin^2\alpha)+(1-\sin^2\beta)+(1-\sin^2\gamma)=1$

$\Rightarrow\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

Example 4: Projections of a line segment on the axes are 12, 4 and 3 respectively. Find the length and direction cosines of the line segment.

Sol: Let ℓ,m,n be the direction cosines and r be the length of the given segment, then $\ell r, mr, nr$ are the projections of the segment on the axes.

Let ℓ, m, n be the direction cosines and r be the length of the given segment, then $\ell r, mr, nr$ are the projections of the segment on the axes; therefore
$\ell r = 12, mr = 4, nr = 3$

Squaring and adding, we get

$$r^2(\ell^2 + m^2 + n^2) = 12^2 + 4^2 + 3^2 \Rightarrow r^2 = 169$$

$\Rightarrow r = 13 \Rightarrow$ length of segment = 13

And direction cosines of segment are
$$\ell = \frac{12}{r} = \frac{12}{13}, m = \frac{4}{r} = \frac{4}{13} \text{ and } n = \frac{3}{r} = \frac{3}{13}$$

Example 5: Find the length of the perpendicular from the point (1, 2, 3) to the line through (6, 7, 7) and having direction ratios (3, 2, –2).

Sol: By using distance formula i.e. $|\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$, we can obtain required length.

Direction cosines of the line are

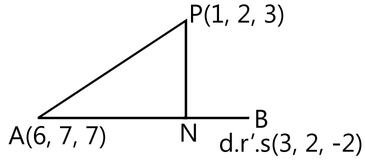
$$\frac{3}{\sqrt{3^2+2^2+(-2)^2}}, \frac{2}{\sqrt{3^2+2^2+(-2)^2}}, \frac{-2}{\sqrt{3^2+2^2+(-2)^2}}$$

i.e. $\frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

$\therefore$ AN = Projection of AP on AB

$$= (6-1)\frac{3}{\sqrt{17}} + (7-2)\frac{2}{\sqrt{17}} + (7-3)\frac{(-2)}{\sqrt{17}}$$

$$= \frac{15+10-8}{\sqrt{17}} = \frac{17}{\sqrt{17}} = \sqrt{17}$$



Also, $AP = \sqrt{(6-1)^2 + (7-2)^2 + (7-3)^2}$

$$= \sqrt{25+25+16} = \sqrt{66}$$

$\therefore PN = \sqrt{AP^2 - AN^2} = \sqrt{66-17} = \sqrt{49} = 7$ unit

Example 6: Find the equation of the plane through the points A(2, 2, –1), B(3, 4, 2) and C(7, 0, 6)

Sol: As we know, equation of a plane passing through the point (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

The general equation of a plane through (2, 2, –1) is
$a(x-2) + b(y-2) + c(z+1) = 0$ .... (i)

It will pass through B(3, 4, 2) and C(7, 0, 6) if

$a(3-2) + b(4-2) + c(2+1) = 0$ or

$a + 2b + 3c = 0$ .... (ii)

& $a(7-2) + b(0-2) + c(6+1) = 0$ or

$5a - 2b + 7c = 0$ .... (iii)

Solving (ii) and (iii) by cross multiplication, we get

$$\frac{a}{14+6} = \frac{b}{15-7} = \frac{c}{-2-10} \quad \text{or} \quad \frac{a}{5} = \frac{b}{2} = \frac{c}{-3} = \lambda \quad \text{(say)}$$

$\Rightarrow a = 5\lambda, b = 2\lambda, c = -3\lambda$

Substituting the values of a, b and c in (i), we get $5\lambda(x-2) + 2\lambda(y-2) - 3\lambda(z+1) = 0$

or, $5(x-2) + 2(y-2) - 3(z+1) = 0$

$\Rightarrow 5x + 2y - 3z = 17$,

Which is the required equation of the plane.

Example 7: Find the angle between the planes x + y + 2z = 9 and 2x – y + z = 15

Sol: By using formula $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

we can obtain the result.

The angle between x + y + 2z = 9 and 2x – y + z = 15 is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{(1)(2) + 1(-1) + (2)(1)}{\sqrt{1^2+1^2+2^2}\sqrt{2^2+(-1)^2+1^2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Example 8: Find the distance between the parallel planes 2x – y + 2z + 3 = 0 and 4x – 2y + 4z + 5 = 0

Sol: By making the coefficient of x, y and z as unity we will be get required result.

Let $P(x_1, y_1, z_1)$ be any point on 2x – y + 2z + 3 = 0, then,

$2x_1 - y_1 + 2z_1 + 3 = 0$ ... (i)

The length of the perpendicular from

$P(x_1, y_1, z_1)$ to $4x - 2y + 4z + 5 = 0$ is

$$= \left|\frac{4x_1 - 2y_1 + 4z_1 + 5}{\sqrt{4^2+(-2)^2+4^2}}\right| = \left|\frac{2(2x_1 - y_1 + 2z_1) + 5}{\sqrt{36}}\right|$$

$$= \left|\frac{2(-3)+5}{6}\right| = \frac{1}{6}$$

Example 9: The equation of a line are 6x – 2 = 3y + 1 = 2z – 2. Find its direction ratios and its equation in symmetric form.

Sol: The given line is 6x – 2 = 3y + 1 = 2z – 2

$$\Rightarrow 6\left(x-\frac{1}{3}\right)=3\left(y+\frac{1}{3}\right)=2(z-1)$$

$$\Rightarrow \frac{x-\frac{1}{3}}{1}=\frac{y+\frac{1}{3}}{2}=\frac{z-1}{3}$$

[We make the coefficients of x, y and z as unity]

This equation is in symmetric form. Thus the direction ratios of the line are 1, 2 and 3 and this line passes through the point $\left(\frac{1}{3},-\frac{1}{3},1\right)$.

Example 10: Find the image of the point (3, –2, 1) in the plane 3x – y + 4z = 2.

Sol: Consider Q be the image of the point P(3, –2, 1) in the plane 3x – y + 4z = 2. Then PQ is normal to the plane hence direction ratios of PQ are 3, –1, 4.

Let Q be the image of the point P(3, –2, 1) in the plane 3x – y + 4z = 2. Then PQ is normal to the plane. Therefore direction ratios of PQ are 3, –1, 4. Since PQ passes through P(3, –2, 1) and has direction ratios 3, –1, 4. Therefore equation of PQ is

$$\frac{x-3}{3}=\frac{y+2}{-1}=\frac{z-1}{4}=r \qquad \text{(say)}$$

Let the coordination of Q be (3r + 3, –r, –2. 4r + 1). Let R be the mid-point of PQ. Then R lies on the plane 3x – y + 4z = 2. The coordinates of R are

$$\left(\frac{3r+3+3}{2},\frac{-r-2-2}{2},\frac{4r+1+1}{2}\right)$$

or $\left(\frac{3r+6}{2},\frac{-r-4}{2},2r+1\right)$

$$3\left(\frac{3r+6}{2}\right)-\left(\frac{-r-4}{2}\right)+4(2r+1)=2$$

$\Rightarrow$ 13r = –13 $\Rightarrow$ r = –1

So, the coordinates of Q are (0, –1, –3)

JEE Advanced/Boards

Example 1: Find the equations of the bisector planes of the angles between the planes 2x – y + 2z + 3 = 0 and 3x – 2y + 6z + 8 = 0 and specify the plane bisecting the acute angle and the plane bisecting obtuse angle.

Sol: As we know, Equation of the planes bisecting the angle between two given planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_1y + c_2z + d_2 = 0$ are

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}=\pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

The two given planes are

2x – y + 2z + 3 = 0 ... (i)

and 3x – 2y + 6z + 8 = 0 ... (ii)

The equations of the planes bisecting the angles between (i) and (ii) are

$$\frac{2x-y+2z+3}{\sqrt{2^2+(-1)^2+2^2}}=\pm\frac{3x-2y+6z+8}{\sqrt{3^2+(-2)^2+6^2}}$$

$$\Rightarrow 14x-7y+14z+21=\pm(9x-6y+18z+24)$$

Hence the two bisector planes are

$5x-y-4z-3=0$ ... (iii)

and $23x-13y+32z+45=0$ ... (iv)

Now we find angle θ between (i) & (iii)

We have,

$$\cos\theta=\frac{5(2)+(-1)(-1)+2(-4)}{\sqrt{2^2+(-1)^2+2^2}\sqrt{5^2+(-1)^2+(-4)^2}}=\frac{1}{\sqrt{42}}$$

Thus the angle between (i) & (iii) is more than $\frac{\pi}{4}$. Therefore, (iii) is the bisector of obtuse angle between (i) and (ii) and hence (iv) bisects acute angle between them.

Example 2: Find the distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to the line whose direction cosines are proportional to 2, 3, –6.

Sol: By using distance formula we can obtain required length,

Equation of line through (1, –2, 3) parallel to the line with d.r.'s 2, 3, –6 is

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=r \qquad \text{...(i)}$$

Any point on it is $(1+2r, -2+3r, 3-6r)$

Line (i) meets the plane x – y + z = 5.

If 1 + 2r – (–2 + 3r) + (3 – 6r) = 5 ; i.e. if $r=\frac{1}{7}$

$\therefore$ Point of intersection is $\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

whose distance from (1, –2, 3) is

$$\sqrt{\left(\frac{9}{7}-1\right)^2+\left(-\frac{11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2}$$

$$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=\sqrt{\frac{49}{49}}=1$$

Example 3: Show that the lines

$$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5} \quad \text{... (i)}$$

and $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ ... (ii)

do not intersect. Also find the shortest distance between them.

Sol: If $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} \neq 0$

then the lines do not intersect each other. And using distance formula we will be get required shortest distance.

Points on (i) and (ii) are (1, –1, 1) and (–2, 1, –1)

respectively and their d.c.'s are $\frac{3}{\sqrt{38}},\frac{2}{\sqrt{38}},\frac{5}{\sqrt{38}}$

and $\frac{4}{\sqrt{29}},\frac{3}{\sqrt{29}},\frac{-2}{\sqrt{29}}$ respectively.

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} -2-1 & 1+1 & -1-1 \\ \frac{3}{\sqrt{38}} & \frac{2}{\sqrt{38}} & \frac{5}{\sqrt{38}} \\ \frac{4}{\sqrt{29}} & \frac{3}{\sqrt{29}} & \frac{-2}{\sqrt{29}} \end{vmatrix}$$

$$=\frac{1}{\sqrt{38}}\times\frac{1}{\sqrt{29}}\begin{vmatrix} -3 & 2 & -2 \\ 3 & 2 & 5 \\ 4 & 3 & -2 \end{vmatrix}$$

$$=\frac{1}{\sqrt{38}}\times\frac{1}{\sqrt{29}},\ [-3(-4-15)+2(20+6)\ -2(9-8)] \neq 0$$

Hence the given lines do not intersect.

Any point P on (i) is $(1 + 3r_1, 2r_1, -1, 5r_1 + 1)$ and a point on (ii) is $Q(4r_2 - 2, 3r_2 + 1, -2r_2 - 1)$

$\therefore$ Direction ratios of PQ are

$(4r_2 - 3r_1 - 3, 3r_2 - 2r_1 + 2, -2r_2 - 5r_1 - 2)$

If PQ is perpendicular to (i) and (ii), we have

$3(4r_2 - 3r_1 - 3) + 2(3r_2 - 2r_1 + 2) + 5(-2r_2 - 5r_1 - 2) = 0$

& $4(4r_2 - 3r_1 - 3) + 3(3r_2 - 2r_1 + 2) - 2(-2r_2 - 5r_1 - 2) = 0$

i.e. $8r_2 - 38r_1 - 15 = 0$ & $29r_2 - 8r_1 - 2 = 0$

Solving them, $\frac{r_2}{76-120}=\frac{r_1}{-435+16}=\frac{1}{1038}$

$$\Rightarrow r_2=-\frac{44}{1038},\ r_1=-\frac{419}{1038}$$

$\therefore$ Points P and Q are $\left(-\frac{1257}{1038}+1,-\frac{838}{1038}-1,-\frac{2095}{1038}+1\right)$

and $\left(-\frac{176}{1038}-2,-\frac{132}{1038}+1,\frac{88}{1038}-1\right)$

We can find the distance PQ by distance formula which is the shortest distance.

Example 4: Find the angle between the lines whose direction ratios satisfy the equations :

$3\ell+m+5n=0,\ 6mn-2n\ell+5\ell m=0$

Sol: Here, $\cos\theta=\frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$.

The given equations are $3\ell+m+5n=0$ 3 ... (i)

and $6mn-2n\ell+5\ell m=0$ ... (ii)

From (i), we have $m=-3\ell-5n$ ... (iii)

Putting in (ii), we get

$6(-3\ell-5n)\,n-2n\ell+5\ell(-3\ell-5n)=0$

$\Rightarrow 30n^2+45\ell n+15\ell^2=0$

$\Rightarrow 2n^2+3\ell n+\ell^2=0 \quad \Rightarrow (n+\ell)(2n+\ell)=0$

$\Rightarrow$ Either $\ell=-n$ or $\ell=-2n$

If $\ell=-n$, then from (iii), $m = -2n$

If $\ell=-2n$, then from (iii), $m = n$

Thus the direction ratios of two lines are

–n, –2n, n and –2n, n, n

i.e. 1, 2, –1 and –2, 1, 1

$\therefore$ If θ is the angle between the lines, then

$$\cos\theta=\frac{1.(-2)+2.1+(-1).1}{\sqrt{1+4+1}.\sqrt{4+1+1}}=\frac{-2+2-1}{\sqrt{6}.\sqrt{6}}=\frac{-1}{6}.$$

Example 5: Find the equation of the plane through the intersection planes 2x + 3y + 4z = 5, 3x – y + 2z = 3 and parallel to the straight line having direction cosines (–1, 1, –1).

Sol: By using formula of family of plane, we will get the result.

Equation of plane through the given planes is $2x+3y+4z-5+\lambda(3x-y+2z-3)=0$

i.e. $(2+3\lambda)x+(3-\lambda)y+(4+2\lambda)z+(-5-3\lambda)=0$

This plane is parallel to the given straight line.

$\Rightarrow -1(2+3\lambda)+1(3-\lambda)+(-1)(4+2\lambda)=0$

$\Rightarrow -2-3\lambda+3-\lambda-4-2\lambda=0$

$\Rightarrow 6\lambda=-3 \qquad \Rightarrow \lambda=-\frac{1}{2}$

$\therefore$ Equation of required plane is

$\frac{1}{2}x+\frac{7}{2}y+3z-\frac{7}{2}=0 \qquad \Rightarrow x+7y+6z=7$

Example 6: Prove that the lines

$$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7} \text{ and } \frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$$

are coplanar. Also, find the plane containing these two lines.

Sol: As similar to example 3.

We know the lines $\frac{x-x_1}{\ell_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$

and $\frac{x-x_2}{\ell_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=0$$

and the equation of the plane containing these two lines is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=0$$

Here $x=-1, y_1=-3, z_1=-5, x_2=2, y_2=4, z_2=6,$

$\ell_1=3, m_1=5, n_1=7, \ell_2=1, m_2=4, n_2=7$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=\begin{vmatrix} 3 & 7 & 11 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$=21-98+77=0$

So, the given lines are coplanar, The equation of the plane containing the lines is

$$=\begin{vmatrix} x+1 & y+3 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}=0$$

or $(x+1)(35-28)-(y+3)(21-7)+(z+5)(12-5)=0$ or $x-2y+z=0$

Example 7: Find the equation of the plane passing through the lines of intersection of the planes 2x – y = 0 and 3z – y = 0 and perpendicular to the plane 4x + 5y – 3z = 8.

Sol: Here by using the family of plane and formula of two perpendicular plane we will get the result.

The plane $2x-y+k(3z-y)=0$

$\Leftrightarrow 2x-(1+k)y+3kz=0$ is perpendicular to the plane $4x+5y-3z=8$

$\Rightarrow 2.4-(1+k).5+3k(-3)=0 \qquad \Rightarrow 14k=3$

$\Rightarrow k=\frac{3}{14}$

Thus the required equation is

$2x-y+\left(\frac{3}{14}\right)(3z-y)=0 \quad \Leftrightarrow 28x-17y+9z=0$

Example 8: Show that the lines

$\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}, 3x+2y+z-2=0=x-3y+2z-13$

are coplanar and find the equation to the plane in which they lie.

Sol: By using the condition of coplanarity of line, we will get given lines are coplanar or not. And after that by using general equation of the plane we can obtain required equation of plane.

The general equation of the plane through the second line is

$3x+2y+z-2+k(x-3y+2z-13)=0$

$\Leftrightarrow x(3+k)+y(2-3k)+z(1+2k)-2-13k=0;$

K being the parameter

This contains the first line only if

$3(3+k)+(2-3k)-2(1+2k)=0 \quad \Rightarrow k=\frac{9}{4}$

Hence the equation of the plane which contains the two lines is

$21x-19y+22x-125=0$

This plane clearly passes through the point $(-5,-4,7)$

JEE Main/Boards

Exercise 1

Q.1 Direction cosines of a line are $\frac{3}{7},\frac{-2}{7},\frac{6}{7}$, find its direction ratios.

Q.2 Find the direction ratios of a line passing through the points (2, 1, 0) and (1, –2, 3).

Q.3 Find the angle between the lines
$\frac{x}{1}=\frac{y}{2}=\frac{z}{0}$ and $\frac{x-1}{3}=\frac{y+5}{2}=\frac{z-3}{1}$.

Q.4 Find the equation of a line parallel to the vector $3\hat{i}-\hat{j}-3\hat{k}$ and passing through the point $(-1,1,1)$.

Q.5 Write the vector equation of a line whose Cartesian equation is $\frac{x+3}{2}=\frac{y-1}{4}=\frac{z+1}{5}$.

Q.6 Write the Cartesian equation of a line whose vector equations is $\vec{r}=(3\hat{i}+2\hat{j}-5\hat{k})+\lambda(-2\hat{i}+\hat{j}+3\hat{k})$.

Q.7 Find the value of p, such that the line
$\frac{x}{1}=\frac{y}{3}=\frac{z}{2p}$ and $\frac{x}{-3}=\frac{y}{5}=\frac{z}{2}$
are perpendicular to each other.

Q.8 Write the Cartesian equation of the plane $\vec{r}(3\hat{i}+2\hat{j}+5\hat{k})=7$.

Q.9 Write the vector equation of plane $3x-y-4z+7=0$.

Q.10 Find the vector, normal to the plane $\vec{r}.(3\hat{i}-7\hat{k})+5=0$.

Q.11 Find the direction ratios of a line, normal to the plane 7x + y – 2z = 1.

Q.12 Find the angle between the line $\frac{x+2}{4}=\frac{y-1}{-5}=\frac{z}{7}$ and the plane $3x-2z+4=0$.

Q.13 Find the distance of the plane x + y + 3z + 7 = 0 from origin.

Q.14 Find the distance of the plane 3x – 3y + 3z = 0 from the point (1, 1, 1).

Q.15 Find the intercepts cut by the plane 3x – 2y + 4z – 12 = 0 on axes.

Q.16 Direction ratios of a line are 1, 3, –2. Find its direction cosines.

Q.17 Find the direction cosines of y-axis.

Q.18 Find the direction ratio of the line
$\frac{x+2}{1}=\frac{2y-1}{3}=\frac{3-z}{5}$.

Q.19 Find the angle between the planes

$r.(\hat{i}-2\hat{j}-2\hat{k})=1$ and $r.(3\hat{i}-6\hat{j}+2\hat{k})=0$.

Q.20 Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(3\hat{i}-6\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})+3$

Q.21 Find the direction cosines of the two lines which are connected by the relations $\ell-5m+3n=0$ and $7\ell^2+5m^2-3n^2=0$.

Q.22 Prove that, the line passing through the point (1, 2, 3) and (–1, –2, –3) is perpendicular to the line passing through the points (–2, 1, 5) and (3, 3, 2).

Q.23 Find the coordinates of the foot of the perpendicular drawn from the point (1, 2, 1) to the line joining the points (1, 4, 6) and (5, 4, 4).

Q.24 If a variable line in two adjacent positions has direction cosines ℓ, m, n and $\ell+\delta\ell, m+\delta m, n+\delta n$, prove that the small angle $\delta\theta$ between two position is given by $(\delta\theta)^2=(\delta\ell)^2+(\delta m)^2+(\delta n)^2$.

Q.25 Verify that $\frac{\ell_1+\ell_2+\ell_3}{\sqrt{3}},\frac{m_1+m_2+m_3}{\sqrt{3}},\frac{n_1+n_2+n_3}{\sqrt{3}}$ can be taken as direction cosines of a line equally inclined to three mutually perpendicular lines with direction cosines $\ell_1,m_1,n_1;\ell_2,m_2,n_2$ and ℓ_3,m_3,n_3

Q.26 Find the equations of line through the point (3, 0, 1) and parallel to the planes x + 2y = 0 and 3y – z = 0.

Q.27 Find the equations of the planes through the intersection of the planes x + 3y + 6 = 0 and 3x – y – 4z = 0 whose perpendicular distance from the origin is equal to 1.

Q.28 Find the equation of the plane through the points (–1, 1, 1) and (1, –1, 1) and perpendicular to the plane x + 2y + 2z = 5.

Q.29 Find the distance of the point (–1, –5, –10) from the plane x – y + z = 5 measured parallel to the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}.$$

Q.30 Find the vector and Cartesian forms of the equation of the plane passing through (1, 2, –4) and parallel to the line $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=\hat{i}-3\hat{j}+5\hat{k}+\mu(\hat{i}+\hat{j}-\hat{k})$.

Q.31 If straight line having direction cosines given by $a\ell+bm+cn=0$ and $fmn+gn\ell+h\ell m=0$ are perpendicular, then prove that $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$.

Q.32 Prove that, the lines x = ay + b, z = cy + d and x = a′y + b′, z = c′y + d′ are perpendicular to each other, if aa′ + cc′ + 1 = 0.

Q.33 Find the equation of the plane passing through the intersection of the planes 4x – y + z = 10 and x + y – z = 4 and parallel to the line with direction ratios 2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

Q.34 The foot of the perpendicular drawn from the origin to the plane is (2, 5, 7). Find the equation of plane.

Q.35 Find the equation of a plane through (–1, –1, 2) and perpendicular to the planes 3x + 2y – 3z = 1 and 5x – 4y + z = 5.

Q.36 Find the angle between the lines whose direction cosines are given by equations $\ell+m+n=0$; $\ell^2+m^2-n^2=0$

Q.37 Find the equation of the line which passes through (5, –7, –3) and is parallel to the line of intersection of the planes x – 3y – 5 = 0 and 9y – z + 16 = 0.

Q.38 Prove that, the plane through the points (1, 1, 1), (1, –1, 1) and (–7, 3, –5) is perpendicular to xz-plane.

Q.39 Find the length and coordinates of the foot of perpendicular from points (1, 1, 2) to the plane 2x – 2y + 4z + 5 = 0.

Q.40 Find the vector equation in the scalar product form, of the plane passing through the points (1, 0, –1), (3, 2, 2) and parallel to line

$$r=\hat{i}+\hat{j}+\lambda(\hat{i}-2\hat{j}+3\hat{k}).$$

Q.41 Find the distance between the parallel planes 2x – y + 3z – 4 = 0 and 6x – 3y + 9z + 13 =0.

Q.42 Prove that, the equation of a plane. Which meets the axes in A, B, and C and the given centroid of triangle ABC is the point (α,β,γ), is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$.

Q.43 Find the equation of the plane passing through the origin and the line of intersection of the planes x – 2y + 3z + 4 = 0 and x – y + z + 3 = 0.

Q.44 Prove that, the line 2x + 2y – z – 6 = 0, 2x + 3y – z – 8 = 0 is parallel to the plane y = 0. Find the coordinates of the point where this line meets the plane x = 0.

Q.45 Find the equation of the plane through the line ax + by + cz + d = 0, a′x + b′y + c′z + d′ = 0 and parallel to the line $\frac{x}{\ell}=\frac{y}{m}=\frac{z}{n}$.

Q.46 Find the equation of a plane parallel to x-axis and has intercepts 5 and 7 on y and z-axis, respectively.

Q.47 A variable plane at a constant distance p from origin meets the coordinate axes in points A, B and C, respectively. Through these points, planes are drawn parallel to the coordinate planes, prove that locus of point of intersection is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

Q.48 Find the value of λ, for which the points with position vectors $\hat{i}-\hat{j}+3\hat{k}$ and $3\hat{i}+\lambda\hat{j}+3\hat{k}$ are equidistant from the plane $\vec{r}.(5\hat{i}+2\hat{j}-7\hat{k})+9=0$.

Q.49 Find the equation of a plane which is at a distance of 7 units from the origin and which is normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$

Q.50 Find the vector equation of the plane, $r=\hat{i}-\hat{j}+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(4\hat{i}-2\hat{j}+3\hat{k})$ in the scalar product from.

Exercise 2

Single Correct Choice Type

Q.1 The sum of the squares of direction cosines of a straight line is

(A) Zero (B) Two

(C) 1 (D) None of these

Q.2 Which one of the following is best condition for the plane ax + by + cz + d = 0 to intersect the x and y axes at equal angle

(A) $|a| = |b|$ (B) $a = -b$

(C) $a = b$ (D) $a^2 + b^2 = 1$

Q.3 The equation of a straight line parallel to the x-axis is given by

(A) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{1}$ (B) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$

(C) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$ (D) $\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$

Q.4 A straight line is inclined to the axes of x and z at angles 45° and 60° respectively, then the inclination of the line to the y-axis is

(A) 30° (B) 45° (C) 60° (D) 90°

Q.5 The coordinates of the point of intersection of the line $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z+2}{-2}$ with the plane $3x + 4y + 5z = 5$

(A) (5, 15, –14) (B) (3, 4, 5)

(C) (1, 3, –2) (D) (3, 12, –10)

Q.6 Perpendicular is drawn from the point (0, 3, 4) to the plane 2x – 2y + z = 10. The coordinates of the foot of the perpendicular are

(A) $\left|-\frac{8}{3},\frac{1}{3},\frac{16}{3}\right|$ (B) $\left|\frac{8}{3},\frac{1}{3},\frac{16}{3}\right|$

(C) $\left|\frac{8}{3},-\frac{1}{3},\frac{16}{3}\right|$ (D) $\left|\frac{8}{3},\frac{1}{3},-\frac{16}{3}\right|$

Q.7 The equation of the plane through the line of intersection of the planes 2x + y – z – 4 = 0 and 3x + 5z – 4 = 0 which cuts off equal intercepts from the x-axis and y-axis is

(A) 3x + 3y – 8z + 8 = 0 (B) 3x + 3y – 8z – 8 = 0

(C) 3x – 3y – 8z – 8 = 0 (D) x + y – 8z – 8 = 0

Q.8 The symmetric form of the equation of the line x + y – z = 1, 2x – 3y + z = 2 is

(A) $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$ (B) $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{5}$

(C) $\frac{x}{2}=\frac{y-1}{3}=\frac{z}{5}$ (D) $\frac{x-1}{2}=\frac{y}{3}=\frac{z}{5}$

Q.9 The line $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ is parallel to the plane

(A) 2x + y + 2z + 3 = 0 (B) 2x – y – 2z = 3

(C) 21x – 12y + z = 0 (D) 2x + y – 2z = 0

Q.10 The vertices of the triangle PQR are (2, 1, 1), (3, 1, 2) and (–4, 0, 1). The area of the triangle is

(A) $\frac{\sqrt{38}}{2}$ (B) $\sqrt{38}$ (C) 4 (D) 2

Q.11 Equation of straight line which passes through the point P(1, 0, –3) and Q(–2, 1, –4) is

(A) $\frac{x-2}{-3}=\frac{y+1}{1}=\frac{z-4}{-1}$ (B) $\frac{x-1}{3}=\frac{y}{1}=\frac{z+3}{1}$

(C) $\frac{x-1/2}{-3}=\frac{y-1}{1}=\frac{z+4}{-1}$ (D) $\frac{x-1}{-3}=\frac{y}{1}=\frac{z+3}{-1}$

Q.12 A point moves so that the sum of the squares of its distances from the six faces of a cube given by x = ±1, y = ±1, z = ±1 is 10 units. The locus of the point is

(A) $x^2 + y + z^2 = 1$ (B) $x^2 + y^2 + z^2 = 2$

(C) x + y + z = 1 (D) x + y + z = 2

Q.13 The points (0, –1, –1), (–4, 4, 4), (4, 5, 1) and (3, 9, 4) are

(A) Collinear (B) Coplanar

(C) Forming a square (D) None of these

Q.14 The equation of the plane containing the line $\frac{x-\alpha}{\ell}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is $a(x-\alpha)+b(y-\beta)+c(z-\gamma)=0$, where $a\ell+bm+cn$ is equal to

(A) 1 (B) –1 (C) 2 (D) 0

Q.15 The reflection of the plane 2 x + 3 y + 4 z − 3 = 0 in the plane x − y + z − 3 = 0 is the plane

(a) 4x− 3y + 2z −15 = 0 (b) x− 3 y + 2z − 15 = 0

(c) 4x + 3y− 2z + 15 = 0 (d) None of these

Previous Years' Questions

Q.1 The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane
$2x-4y+z=7$, is ***(2003)***

(A) 7 (B) –7 (C) No real value (D) 4

Q.2 If the lines $\vec{r}=\vec{a}_2+\mu\vec{b}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is ***(2004)***

(A) $\vec{a}_2$ (B) $\frac{9}{2}$ (C) $-\frac{2}{9}$ (D) $-\frac{3}{2}$

Q.3 A variable plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ at a unit distance from origin cuts the coordinate axes at A, B and C. Centroid (x, y, z) satisfies the equation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=K$. The value of K is ***(2005)***

(A) 9 (B) 3 (C) 1/9 (D) 1/3

Fill in the Blanks for Q.4 and Q.5

Q.4 The area of the triangle whose vertices are A(1, –1, 2), B(2, 1, –1), C(3, –1, 2) is … ***(1983)***

Q.5 The unit vector perpendicular to the plane determined by P(1, –1, 2), Q(2, 0, –1) and R(0, 2, 1) is…………. ***(1983)***

Q.6 A plane is parallel to two lines whose direction ratios are (1, 0, –1) and (–1, 1, 0) and it contains the point (1, 1, 1). If it cuts coordinate axes at A, B, C. Then find the volume of the tetrahedron OABC. ***(2004)***

Q.7 Find the equation of the plane containing the line 2x – y + z – 3 = 0, 3x + y + z = 5 and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). ***(2005)***

Q.8 If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $\ell x+my-z=9$, then ℓ^2+m^2 is equal to: ***(2016)***

(A) 18 (B) 5 (C) 2 (D) 26

Q.9 The distance of the point $(1,-5,9)$ from the plane $x-y+z=5$ measured along the line $x=y=z$ is ***(2016)***

(A) $10\sqrt{3}$ (B) $\frac{10}{\sqrt{3}}$ (C) $\frac{20}{3}$ (D) $3\sqrt{10}$

Q.10 The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}-\frac{y+1}{4}-\frac{z-2}{12}$ and the plane $x-y+z=16$, is: ***(2015)***

(A) 8 (B) $3\sqrt{21}$ (C) 13 (D) $2\sqrt{14}$

Q.11 The equation of the plane containing the line $2x-5y+z+3; x+y+4z=5$ and parallel to the plane $x+3y+6z=1$ is ***(2015)***

(A) $x+3y+6z=-7$ (B) $x+3y+6z=7$

(C) $2x+6y+12z=-13$ (D) $2x+6y+12z=13$

Q.12 The number of common tangents to the circles ***(2015)***

(A) Meats the curve again in the second in the second quadrant

(B) Meats the curve again in the third quadrant

(C) Meets the curve again in the fourth quadrant

(D) Does not meet the curve again

Q.13 The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3$ is the ***(2014)***

(A) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$ (B) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z-2}{5}$

(C) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$ (D) $\frac{x-3}{-3}=\frac{y-5}{-1}=\frac{z-2}{-5}$

Q.14 The angle between the lines whose direction cosines satisfy the equations l+m+n=0 and $l^2=m^2+n^2$ is ***(2014)***

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Q.15 If the lines

$\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$

are coplanar, then k have ***(2013)***

(A) Exactly one value (B) Exactly two value

(C) Exactly three values (D) Any value

Q.16 An equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is ***(2012)***

(A) $x-2y+2z-3=0$ (B) $x-2y+2z+1=0$

(C) $x-2y+2z-1=0$ (D) $x-2y+2z+5=0$

Q.17 If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals ***(2011)***

(A) $\frac{3}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{3}$ (D) $\frac{2}{3}$

Q.18 Statement–I: The point $A(1,0,7)$ is the mirror image of the point $B(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Statement-II: The line: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining $A(1,0,7)$ and $B(1,6,3)$ ***(2011)***

(A) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(B) Statement-I is true, statement-II is false.

(C) Statement-I is false, statement-II is true

(D) Statement-I is true, statement-II is true, statement-II is a correct explanation for statement-I

JEE Advanced/Boards

Exercise 1

Q.1 Points X and Y are taken on the sides QR and RS respectively, of parallelogram PQRS, so that $QX = 4\overrightarrow{XR}$ and $\overrightarrow{RY} = 4\overrightarrow{YS}$. The line XY cuts the line PR at Z. prove that $\overrightarrow{PZ} = \left(\frac{21}{25}\right)\overrightarrow{PR}$.

Q.2 Given three points on the xy plane on O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $(\overrightarrow{PA}\cdot\overrightarrow{PB}) + 3(\overrightarrow{OA}\cdot\overrightarrow{OB}) = 0$. If the maximum and minimum values of $|\overrightarrow{PA}||\overrightarrow{PB}|$ are M and m, respectively then find the value of $M^2 + m^2$.

Instruction for questions 3 to 6.

Suppose the three vectors, $\vec{a},\vec{b},\vec{c}$ on a plane satisfy the condition that

$|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{a}+\vec{b}| = 1; c$ is perpendicular to a and $\vec{b}\cdot\vec{c} > 0$, then

Q.3 Find the angle formed by $2\vec{a}+\vec{b}$ and $\vec{b}$.

Q.4 If the vector c is expressed as a linear combination $\lambda\vec{a}+\mu\vec{b}$ then find the ordered pair

$\frac{\ell_1-\ell_2}{2\sin(\theta/2)}, \frac{m_1-m_2}{2\sin(\theta/2)}$ and $\frac{n_1-n_2}{2\sin(\theta/2)}$.

Q.5 For real number x, y the vector $\vec{p} = x\vec{a} + y\vec{c}$ satisfies the condition $0 \le \vec{p}\cdot\vec{a} \le 1$ and $0 \le \vec{p}\cdot\vec{b} \le 1$. Find the maximum value of $\vec{p}.\vec{c}$

Q.6 For the maximum value of x and y, find the linear combination of $\vec{p}$ in terms of $\vec{a}$ and $\vec{b}$.

Q.7 If O be the origin and the coordinates of P be (1, 2, -3), then find the equation of the plane passing through P and perpendicular to OP.

Q.8 Given non zero number x_1, x_2, x_3; y_1, y_2, y_3 and z_1, z_2 and z_3 (i) Can the given numbers satisfy

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0 \text{ and } \begin{cases} x_1x_2 + y_1y_2 + z_1z_2 = 0 \\ x_2x_3 + y_2y_3 + z_2z_3 = 0 \\ x_3x_1 + y_3y_1 + z_3z_1 = 0 \end{cases}$$

(ii) If $x_1 > 0$ and $y_1 < 0$ for all I = 1, 2, 3 and $P = (x_1, x_2, x_3)$; $Q(y_1, y_2, y_3)$ and O(0, 0, 0) can the triangle POQ be a right angled triangle?

Q.9 ABCD is a tetrahedron with pv's of its angular points as $A(-5,22,5); B(1,2,3); C(4,3,2)$ and $D(-1,2,-3)$. If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelogram is S, then find the value of S.

Q.10 If x, y are two non-zero and non-collinear vectors satisfying $[(a-2)\alpha^2+(b-3)\alpha+c]x+[(a-2)\beta^2+(b-3)\beta+c]y$ $+[(a-2)\gamma^2+(b-3)\gamma+c](x\times y)=0$

where α,β,γ are three distinct real numbers, then find the value of $(a^2+b^2+c^2)$.

Q.11 Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Q.12 Find the equations of the straight line passing through the point (1, 2, 3) to intersect the straight line $x+1=2(y-2)=x+4$ and parallel to the plane $x+5y+4z=0$.

Q.13 Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

Exercise 2

Single Correct Choice Type

Q.1 If P(2, 3, –6) and Q(3, –4, 5) are two points, the direction cosines of line PQ are

(A) $-\frac{1}{\sqrt{171}},-\frac{7}{\sqrt{171}},-\frac{11}{\sqrt{171}}$ (B) $\frac{1}{\sqrt{171}},-\frac{7}{\sqrt{171}},\frac{11}{\sqrt{171}}$

(C) $\frac{1}{\sqrt{171}},\frac{7}{\sqrt{171}},-\frac{11}{\sqrt{171}}$ (D) $-\frac{1}{\sqrt{171}},-\frac{7}{\sqrt{171}},\frac{11}{\sqrt{171}}$

Q.2 The ratio in which yz-plane divide the line joining the points A(3, 1, –5) and B(1, 4, –6) is

(A) –3 : 1 (B) 3 : 1 (C) –1 : 3 (D) 1 : 3

Q.3 The value of λ for which the lines $3x+2y+z+5=0=x+y-2z-3$ and $2x-y-\lambda z=0$ $=7x+10y-8z$ are perpendicular to each other is

(A) –1 (B) –2 (C) 2 (D) 1

Q.4 The ratio in which yz-plane divides the line joining (2, 4, 5) and (3, 5, 7)

(A) –2 : 3 (B) 2 : 3 (C) 3 : 2 (D) –3 : 2

Q.5 A line makes angle $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$ is equal to

(A) 1 (B) 4/3 (C) ¾ (D) 4/5

Q.6 A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C. The locus of the point common to plane through A, B, C parallel to coordinate planes is

(A) ayz + bzx + cxy = xyz (B) axy + byz + czx = xyz

(C) axy + byz + czx = abc (D) bcx + acy + abz = abc

Q.7 The equation of the plane bisecting the acute angle between the planes

2x – y + 2z + 3 = 0 and 3x – 2y + 6z + 8 = 0

(A) 23x – 13y + 32z + 45 = 0

(B) 5x – y – 4z = 3

(C) 5x – y – 4z + 45 = 0

(D) 23x – 13y + 32z + 3 = 0

Q.8 The shortest distance between the two straight lines $\frac{x-4/3}{2}=\frac{y+6/5}{3}=\frac{z-3/2}{4}$ and $\frac{5y+6}{8}=\frac{2z-3}{9}=\frac{3x-4}{5}$ is

(A) $\sqrt{29}$ (B) 3 (C) 0 (D) $6\sqrt{10}$

Q.9 The equation of the straight line through the origin parallel to the line (b + c)x + (c + a)y + (a + b) z = k = (b – c)x + (c – a)y + (a – b)z is

(A) $\frac{x}{b^2-c^2}=\frac{y}{c^2-a^2}=\frac{z}{a^2-b^2}$

(B) $\frac{x}{b}=\frac{y}{c}=\frac{z}{a}$

(C) $\frac{x}{a^2-bc}=\frac{y}{b^2-ca}=\frac{z}{c^2-ab}$

(D) None of these

Assertion Reasoning Type

Q.10 Consider the following statements

Assertion: The plane y + z + 1 = 0 is parallel to x-axis.

Reason: Normal to the plane is parallel to x-axis.

(A) Both A and R are true and R is the correct

(B) Both A and R are true and R is not a correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Previous Years' Questions

Q.1 A plane passes through (1, –2, 1) and is perpendicular to two planes 2x – 2y + z = 0 and x – y + 2z = 4, then the distance of the plane from the point (1, 2, 2) is ***(2006)***

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Q.2 Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x – 4y + 3z = 1 is ***(2009)***

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Q.3 A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals ***(2009)***

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

For the following question, choose the correct answer from the codes (A), (B), (C) and (D) defined as follows.

(A) Statement-I is true, statement-II is also true; statement-II is the correct explanation of statement-I

(B) Statement-I is true, statement-II is also true; statement-II is not the correct explanation of statement-I.

(C) Statement-I is true; statement-II is false.

(D) Statement-I is false; statement-II is true

Q.4 Consider the planes 3x – 6y – 2z = 15 and 2x + y –2z = 5.

Statement-I: The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z =15t.

Statement-II: The vectors $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of the given planes. ***(2007)***

Q.5 Consider three planes

$$AB: \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda$$

$$CD: \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} = \mu \text{ and}$$

$$L \equiv (3\lambda+3, -\lambda+8, \lambda+3)$$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

Statement-I : At least two of the lines L_1, L_2 and L_3 are non-parallel.

Statement-II : The three planes do not have a common point ***(2008)***

Paragraph for Q.6 to Q.8

Read the following passage and answer the questions. Consider the lines

$$L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}, L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

(2008)

Q.6 The unit vector perpendicular to both L_1 and L_2 is

(A) $\frac{-\hat{i} + 7\hat{j} + 7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

(C) $\frac{-\hat{i} + 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i} - 7\hat{j} - \hat{k}}{\sqrt{99}}$

Q.7 The shortest distance between L_1 and L_2 is

(A) 0 (B) $\frac{17}{\sqrt{3}}$ (C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

Q.8 The distance of the point (1, 1, 1) from the plane passing through the point (-1, -2, -1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$ (C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

Match the Columns

Match the condition/expression in column I with statement in column II.

Q.9 Consider the following linear equations ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz =0 ***(2007)***

Column I	Column II
(A) $a+b+c \neq 0$ and $a^2+b^2+c^2 = ab+bc+ca$	(p) The equations represent planes meeting only at a single point

(B) $a+b+c=0$ and $a^2+b^2+c^2 \neq ab+bc+ca$	(q) The equation represent the line x = y = z
(C) $a+b+c\neq 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$	(r) The equations represent identical planes
(D) $a+b+c=0$ and $a^2+b^2+c^2 \; ab+bc+ca$	(s) The equations represent the whole of the three dimensional space

Q.10 (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6), then the point Q such that PQ is perpendicular to the plane in (a) and the mid point of PQ lies on it. ***(2003)***

Q.11 T is a parallelepiped in which A, B, C and D are vertices of one face and the face just above it has corresponding vertices A', B', C', D', T is now compressed to S with face ABCD remaining same and A', B', C', D' shifted to A", B", C", D" in S. the volume of parallelepiped S is reduced to 90% of T. Prove that locus of A" is a plane. ***(2003)***

Q.12 Consider a pyramid OPQRS located in the first octant $(x\geq 0, y\geq 0, z\geq 0)$ with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with $OP=3$. The point S is directly above the mid-point T of diagonal OQ such that $TS=3$. Then ***(2016)***

(A) The acute angle between OQ and OS is $\frac{\pi}{3}$

(B) The equation of the plane containing the triangle OQS is $x-y=0$

(C) The length of the perpendicular from p to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) The perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Q.13 Let P be the image of the point $(3,1,7)$ with respect to the plane $x-y+x=3$. Then equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is ***(2016)***

(A) $x+y-3z=0$ (B) $3x+z=0$

(C) $x-4y+7z=0$ (D) $2x-y=0$

Q.14 In R^3, consider the planes $P_1: y=0$ and $P_2: x+z=1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the distance of the point (0, 1, 0) from P_3 is 1 and the distance a point (α, β, γ) from p_3 is 2, then which of the following relations is (are) true? ***(2015)***

(A) $2\alpha+\beta+2\gamma+2=0$ (B) $2\alpha-\beta+2\gamma+4=0$

(C) $2\alpha+\beta-2\gamma-10=0$ (D) $2\alpha-\beta+2\gamma-8=0$

Q.15 In R^3 let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? ***(2015)***

(A) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (B) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$

(C) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (D) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

Q.16 From a point $p(\lambda,\lambda,\lambda)$ perpendiculars PQ and PR are drawn respectively on the lines $y=x, z=1$ and $y=-x, z=-1$. If p is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are) ***(2014)***

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

Q.17 Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane $x+y+z=3$. The feet of perpendiculars lie on the line **(2013)**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

Q.18 Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar.

The α can take value(s) ***(2013)***

(A) 1 (B) 2 (C) 3 (D) 4

Q.19 Consider the lines $L_1: \frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}$, $L_2: \frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$ and the planes

$P_1 : 7x + y + 2z = 3, P_2 : 3x + 5y - 6z = 4.$

Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2.and perpendicular to planes P_1 and P_2

Match List I with List II and select the correct answer using the code given below the list: ***(2013)***

List I	List II
p. a=	1. 13
q. b=	2. -3
r. c=	3. 1
s. d=	4. -2

Codes:

	p	q	r	s
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

Q.20 The point P is the intersection of the straight line joining the point $Q(2,3,5)$ and $R(1,-1,4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2,1,4)$ to QR, then the length of the line segment PS is ***(2012)***

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Q.21 The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$ is ***(2012)***

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

Q.22 If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0,\infty)$ then ***(2012)***

(A) f has a local maximum at $x = 2$

(B) f is decreasing on (2, 3)

(C) There exists some $c \in (0,\infty)$ such that f′(c)=0

(D) f has local minimum at $x = 3$

Q.23 If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

(2010)

Q.24 If the distance of the point $P(1,-2,1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is ***(2010)***

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Q.25 Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD′ makes a right angle with the side AB, then the cosine of the angle a is given by ***(2010)***

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.5 Q.10 Q.23
Q.29 Q.36 Q.40
Q.42 Q.47 Q.49
Q.50

Exercise 2

Q.2 Q.8 Q.12
Q.13 Q.14

Previous Years' Questions

Q.3 Q.6

JEE Advanced/Boards

Exercise 1

Q.2 Q.5 Q.8
Q.10 Q.13

Exercise 2

Q.2 Q.5 Q.6
Q.7 Q.9

Previous Years' Questions

Q.3 Q.5 Q.6
Q.9 Q.11

Answer Key

JEE Main/Boards

Exercise 1

Q.1 <3, –2, 6>

Q.2 <1, 3, –3>

Q.3 $\cos^{-1}\left(\frac{7}{\sqrt{70}}\right)$

Q.4 $r = (-\hat{i} + \hat{j} + \hat{k}) + \lambda(3\hat{i} - \hat{j} - 3\hat{k})$

Q.5 $r = (-3\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 5\hat{k})$

Q.6 $\frac{x-3}{-2} = \frac{y-2}{1} = \frac{z+5}{3}$

Q.7 –3

Q.8 3x + 2y +5z = 7

Q.9 $r(3\hat{i} - \hat{j} - 4\hat{k}) + 7 = 0$

Q.10 $3\hat{i} - 7\hat{j}$

Q.11 <7, 1, –2>

Q.12 $\sin^{-1}\left(\frac{-2}{\sqrt{90}\sqrt{13}}\right)$

Q.13 $\frac{7}{\sqrt{11}}$

Q.14 $\frac{1}{\sqrt{3}}$

Q.15 4, –6, 3

Q.16 $\left(\frac{1}{\sqrt{14}} \frac{3}{\sqrt{14}} \frac{-2}{\sqrt{14}}\right)$

Q.17 <0, 1, 0>

Q.18 <2, 3, –10>

Q.19 $\cos^{-1}\left(\frac{11}{21}\right)$ **Q.20** $\sin^{-1}\left(\frac{-1}{7\sqrt{3}}\right)$ **Q.21** $\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right),\left(\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}\right)$

Q.23 (3, 4, 5) **Q.26** $\frac{x-3}{-2}=\frac{y}{1}=\frac{z-1}{3}$ **Q.27** $x-2y-2x-3=0;\ 2x+y-2z+3=0$

Q.28 $2x+2y-3z+3=0$ **Q.29** 13 units **Q.30** $9x-8y+z+11=0$

Q.33 $5y-5z-6=0, \frac{3\sqrt{2}}{5}$ **Q.34** $2x+5y+7z=78$ **Q.35** $5x+9y+11z-8=0$

Q.36 $\frac{\pi}{4}$ **Q.37** $\frac{x-5}{3}=\frac{y+7}{1}=\frac{z+3}{9}$ **Q.39** $\left(-\frac{1}{12},\frac{25}{12},-\frac{1}{6}\right),\frac{13\sqrt{6}}{12}$

Q.40 $r(4\hat{i}-\hat{j}-2\hat{k})=6$ **Q.41** $\frac{25\sqrt{14}}{42}$ **Q.43** $x+2y-5z=0$

Q.44 (0, 2, –2) **Q.45** $(ax+by+cz+d)-\frac{a\ell+bm+cn}{(a'1+b'm+c'm)}(a'x+b'y+c'z+a')=0$

Q.46 $7y+5z=35$ **Q.48** $\lambda=3,-6$ **Q.49** $r(3\hat{i}+5\hat{j}-6\hat{k})-7\sqrt{70}=0$

Q.50 $r(5\hat{i}+\hat{j}-6\hat{k})=4$

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** A **Q.3** D **Q.4** C **Q.5** A **Q.6** B

Q.7 B **Q.8** D **Q.9** C **Q.10** A **Q.11** D **Q.12** B

Q.13 B **Q.14** D **Q.15** A

Previous Years' Questions

Q.1 A **Q.2** B **Q.3** A **Q.4** $\sqrt{13}$ sq. units **Q.5** $\pm\frac{(2\hat{i}+\hat{j}+\hat{k})}{\sqrt{6}}$ **Q.6** $\frac{9}{2}$cu unit

Q.7 $2x-y+z-3=0$ and $62x+29y+19z-105=0$ **Q.8** C **Q.9** A **Q.10** C

Q.11 B **Q.12** B **Q.13** A **Q.14** A **Q.15** B **Q.16** A

Q.17 D **Q.18** B

JEE Advanced/Boards

Exercise 1

Q.2 34 **Q.3** $\frac{\pi}{2}$ **Q.4** $\left(\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$ **Q.5** $\sqrt{3}$ **Q.6** $p=2(\vec{a}+\vec{b})$

Q.7 $x+2y-3z=14$ **Q.8** No, No **Q.9** $\sqrt{110}$ **Q.10** 13 **Q.11** 13

Q.12 $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{-3}$ **Q.13** $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ or $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** A **Q.3** D **Q.4** A **Q.5** B **Q.6** A

Q.7 A **Q.8** C **Q.9** C

Assertion Reasoning Type

Q.10 C

Previous Years' Question

Q.1 D **Q.2** A **Q.3** C **Q.4** D **Q.5** D **Q.6** B

Q.7 D **Q.8** C

Q.9 $A \to r; B \to q; C \to p; D \to s$ **Q.10** (a) x + y – 2z = 3 (b) Q(6, 5, –2) **Q.12** B, C, D

Q.13 C **Q.14** B, D **Q.15** A, B **Q.16** C **Q.17** D **Q.18** A, D

Q.19 A **Q.20** A **Q.21** A **Q.22** B, C **Q.23** 6 **Q.24** A

Q.25 B

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $l = \frac{3}{7}$ $m = \frac{-2}{7}$ $n = \frac{6}{7}$

Direction ratios are <3, –2, 6>

Sol 2: [2, 1, 0] & [1, –2, 3]

Direction ratios = 2 – 1, 1 + 2, 0 – 3 = <1, 3, –3>

Sol 3: $\frac{x}{1} = \frac{y}{2} = \frac{z}{0}$ and $\frac{x-1}{3} = \frac{y+5}{2} = \frac{z-3}{1}$

<1, 2, 0> and <3, 2, 1>

$$\cos\theta = \frac{1.3+2.2+0.1}{\sqrt{5}\sqrt{14}} = \frac{7}{\sqrt{70}} \Rightarrow \theta = \cos^{-1}\left(\frac{7}{\sqrt{70}}\right)$$

Sol 4: $\frac{x+1}{3} = \frac{y-1}{-1} = \frac{z-1}{-3} = t$

$r = \hat{i} + \hat{j} + \hat{k} + t\left(3\hat{i} - \hat{j} - 3\hat{k}\right)$

Sol 5: $r = -3\hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} + 4\hat{j} + 5\hat{k})$

Sol 6: $x\hat{i} + y\hat{j} + z\hat{k} = (3 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (-5 + 3\lambda)\hat{k}$

$\frac{x-3}{-2} = \frac{y-2}{1} = \frac{z+5}{3}$

Sol 7: $\cos\theta = 0 = -3.1 + 3.5 + 2p.2$

$\Rightarrow 12 + 4p = 0 \Rightarrow p = -3$

Sol 8: (xi + yj + zk) . (3i + 2j + 5k) = 7

3x+ 2y + 5z = 7

Sol 9: 3x – y – 4z = –7; r(3i – j – 4k) = –7

$r(3\hat{i} - \hat{j} - 4\hat{k}) + 7 = 0$

Sol 10: 3x – 7y = –5

Direction ratios of normal to plane are (3, –7, 0) the vector along that normal is $3\hat{i} - 7\hat{j}$.

Sol 11: 7x + y – 2z = 1

Direction ratios of vector normal to the plane are

7i + j – 2k = 0

(7, 1, -2)

Sol 12: Direction ratios of line <4, -5, 7>

Direction ratio of line perpendicular to plane <3, 0, –2>

$\sin\theta = \frac{4\times 3+(-5)\times(0)+7\times(-2)}{\sqrt{16+25+49}\sqrt{9+0+4}} = \frac{-2}{\sqrt{90}\sqrt{13}}$

Sol 13: x + y + 3z + 7 = 0

Distance from origin is $\frac{0+0+3(0)+7}{\sqrt{1+1+9}} = \frac{7}{\sqrt{11}}$

Sol 14: 3x – 3y + 3z = 0

Distance from (1, 1, 1) is $\frac{3(1)-3(1)+3(1)}{\sqrt{9+9+9}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

Sol 15: 3x – 2y + 4z = 12

Intercept on x-axis (y, z = 0, 0)x = 4

Intercept on y-axis (z, z = 0, 0)y = –6

Intercept on z-axis (x, y = 0, 0)z = 3

Sol 16: <a, b, c> = <1, 3, –2>

$< l, m, n> = \left(\frac{1}{\sqrt{1+9+4}}, \frac{3}{\sqrt{1+9+4}}, \frac{-2}{\sqrt{1+9+4}}\right)$

$= \left(\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}\right)$

Sol 17: Direction cosines of y-axis = <0, 1, 0>

Sol 18: $\frac{x+2}{1} = \frac{2y-1}{3} = \frac{3-z}{5}$

$\Rightarrow \frac{x+2}{1} = \frac{y-\frac{1}{2}}{\frac{3}{2}} = \frac{z-3}{-5}$

Direction ratio are $\left\langle 1, \frac{3}{2}, -5\right\rangle$ or $\langle 2, 3, -10\rangle$

Sol 19: $\vec{r}$. (i – 2j – 2k) = 1; $\vec{r}$. (3i – 6j + 2k) = 0

$\cos\theta = \frac{3.1+(-6).(-2)+(2)(-2)}{\sqrt{1+4+4}\sqrt{9+36+4}} = \frac{3+12-4}{3.7} = \frac{11}{21}$

$\theta = \cos^{-1}\left(\frac{11}{21}\right)$

Sol 20: $\vec{r}$ = 2i – j + 3k + λ(3i – 6j + 2k) and Plane $\vec{r}$. (i + j + k) = 3

$\text{Sin}\theta = \frac{3.1-6.1+2.1}{\sqrt{3}\sqrt{9+36+4}} = \frac{-1}{7\sqrt{3}}$

Sol 21: l - 5m + 3n = 0 ; $7l^2 + 5m^2 - 3n^2 = 0$

$l = 5m - 3n$

$\Rightarrow 7(25m^2 + 9n^2 - 30mn) + 5m^2 - 3n^2 = 0$

$\Rightarrow 180m^2 + 60n^2 - 210mn = 0$

$\Rightarrow 6m^2 - 7mn + 2n^2 = 0$

$\Rightarrow 6m^2 - 4mn - 3mn + 2n^2 = 0$

$\Rightarrow$ 2m (3m – 2n) – n(3m – 2n) = 0

$\Rightarrow m = \frac{n}{2}$ or $m = \frac{2n}{3}$

If $m = \frac{n}{2}$, l = –m, if $m = \frac{2n}{3}$, $l = \frac{m}{2}$

The following ratio are

$\left\langle \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$ or $\left\langle \frac{+1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

Sol 22: Line through the points

$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{6} = \lambda$

$\frac{x-3}{5} = \frac{y-3}{2} = \frac{z-3}{-3} = \lambda$

$\cos\theta = \frac{2.5+4.2+6.(-3)}{\sqrt{56}\sqrt{38}} = 0$

$\theta = 90°$

Sol 23: $\frac{x-5}{4} = \frac{y-4}{0} = \frac{z-4}{-2}$

equation of line $= \lambda$

Let foot of ⊥ is (α, β, γ)

$\alpha = 5 + 4\lambda$; $\beta = 4$; $\gamma = 4 - 2\lambda$

$\Rightarrow (\alpha-1).4 + (\beta-2).0 + (\gamma-1).(-2) = 0$

$\Rightarrow (4+4\lambda)4 - 2(3-2\lambda) = 0 \Rightarrow 20\lambda + 10 = 0 \Rightarrow \lambda = \frac{-1}{2}$

$\Rightarrow \alpha = 5 + 4\left(\frac{-1}{2}\right) = 3$ β = 4 = 4

$\Rightarrow \gamma = 4 - 2\left(\frac{-1}{2}\right) = 5$

(3, 4, 5)

Sol 24: $\cos(\delta\theta)$

$$= \frac{l.(l+\delta l)+m.(m+\delta m)+n.(n+\delta n)}{\sqrt{l^2+m^2+n^2}\sqrt{(l+\delta l)^2+(m+\delta m)^2+(n+\delta n)^2}}$$

$\begin{bmatrix}\text{neglecting}\\ \delta l^2, \delta m^2, \delta n^2\end{bmatrix}$

$$= \frac{l^2+m^2+n^2+l\delta l+m\delta m+n\delta n}{\sqrt{(l^2+m^2+n^2)}\sqrt{(l^2+2l\delta l+m^2+2l\delta m+n^2+2l\delta n)}}$$

$$\frac{1-(\delta\theta)^2}{2} = \frac{1+l\delta l+m\delta m+n\delta n}{1}$$

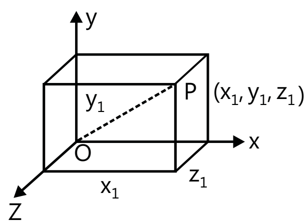
$(\delta\theta)^2 = -2(l\delta l+m\delta m+n\delta n)$ ... (i)

$l^2+m^2+n^2 = (l+\delta l)^2+(m+\delta m)^2+(n+\delta n)^2$

$\Rightarrow (\delta l)^2+(\delta m)^2+(\delta n)^2 = -2l\delta l-2m\delta m-2n\delta n$ ... (ii)

$(\delta\theta)^2 = (\delta l)^2+(\delta m)^2+(\delta m)^2$

Sol 25:



$$\left(\frac{l_1+l_2+l_3}{\sqrt{3}}\right)l_1+\left(\frac{m_1+m_2+m_3}{\sqrt{3}}\right)m_1+\left(\frac{n_1+n_2+n_3}{\sqrt{3}}\right).n_1$$

$$= \frac{1+l_1l_2+l_1l_3+m_1m_2+m_3m_1+n_1m_2+n_1n_3}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Similarly dot product with l_2 and l_3 gives $\frac{1}{\sqrt{3}}$ as result

i.e. it makes same angle with (l_1, m_1, n_1) (l_2, m_2, n_2) and (l_3, m_3, n_3)

Sol 26: $x + 2y = 0$ ... (i)

$3y - z = 0$

$2y - \frac{2z}{3} = 0$ ... (ii)

The line will be across $(a_1, b_1, c_1) \times (a_2, b_2, c_2)$

$(1\ 2\ 0) \times (0\ 3\ -1)$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 3 & -1 \end{vmatrix} = i(-2) - j(-1) + k(3) = -2i + j + 3k$$

Equation of line will be $\frac{x-3}{-2} = \frac{y-0}{1} = \frac{z-1}{3}$

Sol 27: $x + 3y + 6 = 0$, $3x - y - 4z = 0$

$x + 3y + 6 + \lambda(3x - y - 4z) = 0$

$x(1 + 3\lambda) + y(3 - \lambda) + z(-4\lambda) + 6 = 0$

Distance from origin $= \frac{6}{\sqrt{(1+3\lambda)^2+(4\lambda)^2+(3-\lambda)^2}} = 1$

$36 = 1+9\lambda^2+6\lambda+16\lambda^2+9+\lambda^2-6\lambda$

$36 = 26\lambda^2+10$

$\lambda = \pm 1$

Planes are $\Rightarrow 4x + 2y - 4z + 6 = 0$ $(\lambda = 1)$

$-2x+4y+4z+6 = 0$ $(\lambda = -1)$

Sol 28: $ax + by + cz = 1$ ... (i)

$(-1, 1, 1)$ lies on (1)

$\Rightarrow -a + b + c = 1$

$(1, -1, 1)$ lies on (1)

$\Rightarrow +a - b + c = 1 \Rightarrow c = 1$

If $\perp$ to $x+2y+2z=5$

$a \cdot 1 + b \cdot 2 + 2 \cdot c = 0$

$a + 2b = -2$

$a - b = 0$

$a = b = \frac{-2}{3}$

Equation of plane is–$2x - 2y + 3z = 3$.

Sol 29: $P(-1 + r\cos\alpha, -5 + r\cos\beta, -10 + r\cos\gamma)$

are coordinates of point at distance r from $(-1, -5, -10)$ along $\langle\alpha, \beta, \gamma\rangle$

Point P lies on the given plane

$x - y + z = 5$

$-1 + r\cos\alpha + 5 - r\cos\beta + r\cos\gamma - 10 = 5$

$r\cos\alpha - r\cos\beta + r\cos\gamma = 11$

$$r = \frac{11}{\frac{3-4+12}{13}} = \frac{11.13}{11} = 13 \text{ units}$$

Sol 30: $ax + by + cz = 1$

$(1, 2, -4)$

$a + 2b - 4c = 1$ ... (i)

This plane is parallel

$r_1 = i + 2j + 4k + \lambda(2i + 3j + 6k)$

$r_2 = i - 3j + 5k + \lambda(i + j - k)$

$\Rightarrow 2a + 3b + 6c = 0$

$\Rightarrow a + b - c = 0$

$\Rightarrow b = -8c$

$\Rightarrow a = 9c$

$\Rightarrow 9c - 16c - 4c = 1$

$\Rightarrow c = \frac{-1}{11}, b = \frac{+8}{11}, a = \frac{-9}{11}$

Equation of plane is $-9x + 8y - z = 11$ or

$\Rightarrow \vec{r}\,.(-9i + 8j - k) = 11$

Sol 31: $al + bm + cn = 0$ ... (i)

and $fmn + gnl + hlm = 0$ ... (ii)

$\Rightarrow \frac{f}{l} + \frac{g}{m} + \frac{h}{n} = 0$ ... (iii)

Comparing (i) and (iii)

$\frac{a}{f} l^2 = \frac{b}{g} m^2 = \frac{c}{h} n^2 = \lambda$

$\Rightarrow l^2 = \frac{f}{a}\lambda \Rightarrow l = \pm\sqrt{\frac{f}{a}\lambda}$

Similarly

$m^2 = \frac{g}{b}\lambda \Rightarrow m = \pm\sqrt{\frac{g}{h}\lambda}$

$n^2 = \frac{h}{c}\lambda \Rightarrow n = \pm\sqrt{\frac{h}{c}\lambda}$

Since, lines are $\perp$

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$-\frac{f}{a}\lambda - \frac{g}{b}\lambda - \frac{h}{c}\lambda = 0$

$\Rightarrow \lambda\left(\frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right) = 0 \Rightarrow \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

Sol 32: $\frac{x-b}{a} = y = \frac{z-d}{c}$ ... (i)

$\frac{x-b'}{a'} = y = \frac{z-d'}{c'}$ ... (ii)

These 2 are perpendicular if $aa' + cc' + 1 = 0$

Sol 33: $4x - y + z - 10 + \lambda(x + y - z - 4) = 0$

$\Rightarrow x(4+\lambda) + y(-1+\lambda) + z(1-\lambda) = 10 + 4\lambda$

$\Rightarrow (4+\lambda)\cdot 2 + (\lambda - 1)\cdot 1 + (1-\lambda)\cdot 1 = 0$

$\Rightarrow 8 - 1 + 1 + 2\lambda = 0 \Rightarrow \lambda = -4$

$\Rightarrow -5y + 5z = -6$, equation of plane

Distance from (1, 1, 1)

$= \frac{-5+5+6}{\sqrt{25+25}} = \frac{6}{5\sqrt{2}} = \frac{3\sqrt{2}}{5}$

Sol 34: Ratios of line perpendicular to plane is {(2 – 0), (5 – 0), (7 – 0)}

Equation of plane is $2x + 5y + 7z = k$

(2, 5, 7) lies on the plane

$2.2 + 5.5 + 7.7 = k = 78$

$2x + 5y + 7z = 78$

Sol 35: Direction ratios of line $\perp$ to the given planes

$3x + 2y - 3y = 1$; $5x - 4y + z = 5$

$\begin{vmatrix} i & j & k \\ 3 & 2 & -3 \\ 5 & -4 & 1 \end{vmatrix} = i(2 - 12) - j(3 + 15) + k(-12 - 10)$

$= -10i - 18j - 22k$

Plane will be $10x + 18y + 22z = k$

Passes through (–1, –1, 2)

$2.(22) - 28 = k \quad \therefore K = +16$

$10x + 18y + 22z - 16 = 0$

$\Rightarrow 5x + 9y + 11z - 8 = 0$

Sol 36: $l + m + n = 0$ and $l^2 + m^2 = n^2$

$\Rightarrow n = -(l+m)$

$\Rightarrow l^2 + m^2 = (l+m)^2 = l^2 + m^2 = l^2 + m^2 + 2m.n$

$\Rightarrow m.n = 0$

$\Rightarrow m = 0$ or $n = 0$

$\Rightarrow (l, m, n) \equiv \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

$\Rightarrow$ Angle $= \frac{\pi}{4}$

Sol 37:

$\begin{vmatrix} i & j & k \\ 1 & -3 & 0 \\ 0 & +9 & -1 \end{vmatrix} = i(3) - j(-1) + k(9) = 3i + j + 9k$

$\frac{x-5}{3} = \frac{y+7}{1} = \frac{z+3}{9}$

Sol 38: $ax + by + cz = 1$

$a + b + c = 1 \Rightarrow a + c = 1 - b$

$a - b + c = 1 \Rightarrow b = 0$

$-7a + 3b - 5 + 5a = 1$

$b = 6 + 2a/3, a = -3, c = 4$

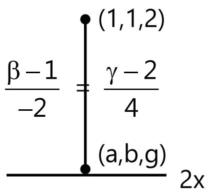
$-3x + 4z = 1 \rightarrow$ ratio $\rightarrow$ [-3, 0, 4]

xz plane $\rightarrow$ ratio $\rightarrow$ [0, 1, 0]

$-3.0 + 0.1 + 4.0 = 0$

Hence given plane is perpendicular to xz plane.

Sol 39: $\frac{\alpha-1}{2} = \frac{\beta-1}{-2} = \frac{\gamma-2}{4}$



$= \frac{-(2-2+8+5)}{4+4+16} = \frac{-13}{24}$

$\alpha = 1 - \frac{13}{12} = -\frac{1}{12}$, $\beta = 1 + \frac{13}{12} = \frac{25}{12}$

$\gamma = 2 - \frac{13}{6} = \frac{-1}{6}$

Length $= \frac{2-2+8+5}{\sqrt{24}} = \frac{13}{\sqrt{24}}$

Sol 40: $ax + by + cz = 1$

$(1, 0, -1) \Rightarrow a - c = 1$

$(3, 2, 2) \Rightarrow 3a + 2b + 2c = 1$

It is parallel to $\langle 1,-2,3 \rangle$

$\Rightarrow a - 2b + 3c = 0$

$\Rightarrow 4a + 5c = 1$

$\Rightarrow 4 + 4c + 5c = 1$

$\Rightarrow c = \frac{-1}{3}$

$\Rightarrow a = \frac{2}{3}$

$\Rightarrow \frac{2}{3} - \frac{3}{3} = +2b$

$b = \frac{-1}{6}$

Eq. of plane $2x - \frac{y}{2} - z = 3$

$4x - y - 2z = 6$

$\Rightarrow \vec{r} \cdot (4\hat{i} - \hat{j} - 2\hat{k}) = 6$

Sol 41: distance between $2x - y + 3z = 4$

$2x - y + 3z = \frac{-13}{3}$

Distance, $d = \frac{4 + \frac{13}{3}}{\sqrt{4+1+9}} = \frac{25}{3\sqrt{14}} = \frac{25\sqrt{14}}{42}$

Sol 42: $ax + by + cz = 1$

$A\left(\frac{1}{a},0,0\right), B\left(0,\frac{1}{b},0\right), C\left(0,0,\frac{1}{c}\right)$

$\frac{1}{a} = 3\alpha$, $\frac{1}{b} = 3\beta$, $\frac{1}{c} = 3\gamma$

$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

Sol 43: $x - 2y + 3z + 4 + \lambda(x - y + z + 3) = 0$

Through origin $3\lambda + 4 = 0$; $\lambda = \frac{-4}{3}$

$\Rightarrow x\left(1 - \frac{4}{3}\right) + y\left(-2 + \frac{4}{3}\right) + z\left(3 - \frac{4}{3}\right) = 0$

$\Rightarrow \vec{r}\, \frac{-x}{3} - \frac{2y}{3} + \frac{5z}{3} = 0$

$\Rightarrow x + 2y - 5z = 0$

Sol 44: $2x + 2y - z - 6 + \lambda(2x + 3y - z - 8) = 0$

$x(2+2\lambda) + y(2+3\lambda) + z(-1-\lambda) - 6 - 8\lambda = 0$ equation of plane

xz plane $\langle 0,1,0 \rangle$ any point on the line is $(\alpha, 2, 2\alpha - 2)$

Direction ratios of line

$$\begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ 2 & 3 & -1 \end{vmatrix} = i(-2+3) - j(-2+2) + k(6-4)$$

$= i + 2k = \langle 1,0,2 \rangle$

This is parallel to plane y = 0 as

(1, 0, 2) . (0, 1, 0) = 0

$\alpha = 0$ i.e. (0, 2, –2)

Sol 45: The equation of Plane

$$ax + by + cz + d + \lambda\left(a'x + b'y + c'z + d'\right) = 0 \qquad \ldots \text{(i)}$$

$$\Rightarrow \left(a + \lambda a'\right)x + \left(b + \lambda b\right)y + \left(c + \lambda c'\right)z + d + \lambda d' = 0$$

Which parallel to line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

$\Rightarrow (a+\lambda a')l+(b+\lambda b')m+(c+\lambda c')n=0$

$\Rightarrow -\frac{al+bm+cn}{a'l+b'm+c'n}=\lambda$

Substituting in (i)

$(ax+by+cz+d)-\frac{al+bm+cn}{a'l+b'm+c'n}(a'x+b'y+c'z+d)=0$

Sol 46: ax + by + cz = 1

$\frac{1}{b}$ = 5, $\frac{1}{c}$ = 7{given intercepts}

<a, b, c> • <1, 0, 0> = 0

a = 0

$\frac{y}{5}+\frac{z}{7}=1$; 7y + 5z = 35

Sol 47: ax + by + cz = 1

$\frac{1}{\sqrt{a^2+b^2+c^2}}$ = P ......(i)

$A\left(\frac{1}{a},0,0\right)$; $B\left(0,\frac{1}{b},0\right)$; $C\left(0,0,\frac{1}{c}\right)$

x = $\frac{1}{a}$, y = $\frac{1}{b}$, c = $\frac{1}{z}$

$\frac{1}{P^2}$ = $a^2+b^2+c^2$ = $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}$ from (i)

Sol 48: i – j + 3k from 5x + 2y – 7z + 9 = 0

$\Rightarrow\left|\frac{5-2-21+9}{\sqrt{49+4+25}}\right|=\frac{9}{\sqrt{78}}$

$\Rightarrow (3i+\lambda j+3k)$ from 5x + 2y – 7z + 9 =0

$\Rightarrow\left|\frac{15+2\lambda-21+9}{\sqrt{49+4+25}}\right|=\left|\frac{3+2\lambda}{\sqrt{78}}\right| \Rightarrow |3+2\lambda|=9$

$\Rightarrow \lambda=3$ or –6

Sol 49: Normal to vector 3i + 5j – 6k

3x +5y – 6z = k

at 7 units from origin

$\left|\frac{k}{\sqrt{36+25+9}}\right|=7$; k = $\pm 7\sqrt{70}$

$\vec{r}\cdot(3\hat{i}-5\hat{j}-6\hat{k})$ = $\pm 7\sqrt{70}$

Sol 50: $r = i-j+\lambda(i+j+k)+\mu(4i-2j+3k)$

$B=\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 4 & -2 & 3\end{vmatrix}$ = i(5) – j(–1) + k(– 2 – 4) = 5i + j – 6 k

Plane pass through (1, –1, 0)

Equation of plane $\vec{r}$ • (5i + j – 6k) = z

5(1) + 1(–1) – 6 (0) – z = 4

The equation of plane ⇒ $\vec{r}$ • (5i + j – 6k) = 4

Exercise 2

Single Correct Choice Type

Sol 1: (C) $l^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

Sol 2: (A) ax + by + cz + d = 0 to intersect x and y axis at equal angle

$|\tan\alpha|=|\tan\beta| \Rightarrow |a|=|b|$

Sol 3: (D) Parallel to x-axis i.e. <1, 0, 0>

$\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$

Sol 4: (C) $\cos\alpha=\frac{1}{\sqrt{2}}$ $\cos\gamma=\cos 60^\circ=\frac{1}{2}$

$l^2+m^2+n^2=1$

$\Rightarrow \frac{1}{2}+\frac{1}{4}+m^2=1 \Rightarrow m^2=\frac{1}{4}$

$\Rightarrow m=\frac{1}{2}$ = $\cos\beta$

$\Rightarrow \beta=60^\circ$

Sol 5: (A) $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z+2}{-2}=\lambda$

$\Rightarrow (-1+\lambda, -3+3\lambda, -2-2\lambda)$

$\Rightarrow 3(-1+\lambda)+4(3\lambda-3)+5(-2-2\lambda)=5$

$\Rightarrow 5\lambda-3-12-10=5 \Rightarrow 5\lambda=30$

⇒ x = 6

(5, 15, –14)

Sol 6: (B)

$\frac{x-0}{2}=\frac{y-3}{-2}=\frac{z-4}{1}=-\frac{(0-6+4-10)}{9}$

$\Rightarrow \frac{x}{2} = \frac{y-3}{-2} = z-4 = \frac{12}{3\times 3} = \frac{4}{3}$

$\Rightarrow x = \frac{8}{3}, y = 3 - \frac{8}{3}, z = 4 + \frac{4}{3}$

$\Rightarrow \frac{8}{3}, \frac{1}{3}, \frac{16}{3}$

Sol 7: (B) $2x + y - z - 4 + \lambda(3x + 5z - 4) = 4$

$2 + 3\lambda = 1 \Rightarrow \lambda = \frac{-1}{3}$

$\Rightarrow 2x - x + y - z - \frac{5}{3}z - 4 + \frac{4}{3} = 0$

$\Rightarrow 3x + 3y - 8z - 8 = 0$

Sol 8: (D) $\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -3 & 1 \end{vmatrix} = i(-2) - j(3) + k(-3-2)$

$= -2i - 3j - 5k$

It passes through (1, 0, 0)

Equation of line is $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{5}$

Sol 9: (C) Line $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ is parallel to plane

$ax + by + cz = 1$

If $a + 2b + 3c = 0$

Only C satisfies the condition

Sol 10: (A) $a = \sqrt{49+1+1} = \sqrt{51}$;

$b = \sqrt{1+0+1} = \sqrt{2}$; $c = \sqrt{36+1+0} = \sqrt{37}$

$s = \frac{\sqrt{2}+\sqrt{51}+\sqrt{37}}{2}$

$s(s-a)(s-b)(s-c)$

$\Rightarrow \frac{\sqrt{51}+\sqrt{37}+\sqrt{2}}{2}\left[\frac{\sqrt{2}+\sqrt{37}-\sqrt{51}}{2}\right]$

$\left[\frac{\sqrt{2}+\sqrt{51}-\sqrt{37}}{2}\right]\left[\frac{\sqrt{37}+\sqrt{51}-\sqrt{2}}{2}\right]$

$= \frac{\left[37+2+2\sqrt{74}-51\right]\left[31-(37+2-2\sqrt{74}\right]}{16}$

$= \frac{\left[2\sqrt{74}-12\right]\left[12+2\sqrt{74}\right]}{16}$

$= \frac{4\times 74-144}{16} = \frac{296-144}{16} = \frac{152}{16} = \frac{38}{4}$

$\Delta = \frac{\sqrt{38}}{2}$

Sol 11: (D) $\frac{x+2}{3} = \frac{y-1}{-1} = \frac{z+4}{1}$

or $\frac{x-1}{3} = \frac{y}{-1} = \frac{z+3}{1}$

Sol 12: (B) Let P(x, y, z) be any point on the locus, then the distances from the six faces are

$|x+1|, |x-1|, |y+1|, |y-1|, |z+1|, |z-1|$

According to the given condition

$|x+1|^2 + |x-1| + |y+1|^2 + |y-1|^2 + |z+1|^2 + |z-1|^2 = 10$

$\Rightarrow \quad 2(x^2 + y^2 + z^2) = 10 - 6 = 4$

$\Rightarrow \quad x^2 + y^2 + z^2 = 2$

Sol 13: (B) If $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ and (x_4, y_4, z_4) are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -4-0 & 4+1 & 4+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix} \Rightarrow = \begin{vmatrix} -4 & 5 & 5 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$= -4(30-20) - 5(20-6) + 5(40-18) = -40 - 70 + 110 = 0$

Sol 14: (D) The plane $y + z + 1 = 0$

Since the plane does not have any intercepts on x-axis, therefore it is parallel to x-axis.

Then normal to plane can not be parallel to x-axis.

Sol 15: (A) Using the fact that reflection of $a'x + b'y + c'z + d' = 0$ in the plane $ax + by + cz + d = 0$ is given by $2(aa' + bb' + cc')(ax + by + cz + d)$

$= (a^2 + b^2 + c^2)(a'x + b'y + c'z + d')$

We get the required equation as

$2(2 + 3 + 4)(x - y + z - 3) = (1 + 1 + 1)(2x - 3y + 4z - 3)$

$6(x - y + z - 3) = 2x - 3y + 4z - 3$

$4x - 3y + 2z - 15 = 0$

Previous Years' Questions

Sol 1: (A) Given equation of straight line

$$\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$$

Since, the line lies in the plane 2x – 4y + z = 7

$\therefore$ Point (4, 2, k) must satisfy the plane.

$\Rightarrow 8 - 8 + k = 7 \Rightarrow k = 7$

Sol 2: (B) Since, the lines intersect they must have a point in common

i.e., $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=\lambda$

and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=\mu$

$\Rightarrow x = 2\lambda+1, y = 3\lambda-1, z = 4\lambda+1$

and $x = \mu+3, y = 2\mu+k, z = \mu$ are same

$\Rightarrow 2\lambda+1 = \mu+3,\ 3\lambda-1 = 2\mu+k,\ 4\lambda+1 = \mu$

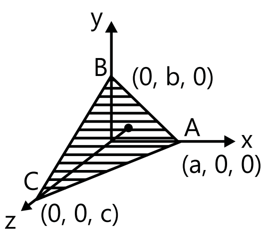
On solving Ist and IIIrd terms, we get,

$\lambda = -\frac{3}{2}$ and $\mu = -5$

$\therefore k = 3\lambda - 2\mu - 1 \Rightarrow k = 3\left(-\frac{3}{2}\right) - 2(-5) - 1 = \frac{9}{2}$

$\therefore k = \frac{9}{2}$

Sol 3: (A) Since, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate axes at A(a, 0, 0), B(0, b, 0), C(0, 0, c) and its distance from origin = 1



$$\therefore \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1$$

or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$ ... (i)

where P is centroid of triangle

$\therefore x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3}$ ... (ii)

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2}=1 \quad \text{or} \quad \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9=K$$

$\therefore K = 9$

Sol 4: Area of $\Delta ABC = \frac{1}{2}(\vec{AB}\times\vec{AC})$, where

$\vec{AB} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{AC} = 2\hat{i}+0\hat{j}+0\hat{k}$

$$\therefore \vec{AB}\times\vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 0 & 0 \end{vmatrix} = 2(-3\hat{j}-2\hat{k})$$

$\Rightarrow$ Area of triangle $= \frac{1}{2}(\vec{AB}\times\vec{AC})$

$= \frac{1}{2}.2.\sqrt{9+4} = \sqrt{13}$ sq. units

Sol 5: A unit vector perpendicular to the plane determined by P, Q, R $= \pm\frac{(\vec{PQ}\times\vec{PR})}{|\vec{PQ}\times\vec{PR}|}$

where $\vec{PQ} = [\hat{i}+\hat{j}]-3\hat{k}$ and $\vec{PR} = -\hat{i}+3\hat{j}-\hat{k}$

$$\therefore \vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$= \hat{i}(-1+9)-\hat{j}(-1-3)+\hat{k}(3+1) = 8\hat{i}+4\hat{j}+4\hat{k}$

$\Rightarrow |\vec{PQ}\times\vec{PR}| = 4\sqrt{4+1+1} = 4\sqrt{6}$

$$\therefore \text{Unit vector} = \pm\frac{(\vec{PQ}\times\vec{PR})}{|\vec{PQ}\times\vec{PR}|} = \pm\frac{4(2\hat{i}+\hat{j}+\hat{k})}{4\sqrt{6}} = \pm\frac{(2\hat{i}+\hat{j}+\hat{k})}{\sqrt{6}}$$

Sol 6: Let the equation of plane through (1, 1, 1) having a, b, c as DR's of normal to plane, $a(x-1)+b(y-1)+c(z-1)=0$ and plane is parallel to straight line having DR's.

(1, 0, –1) and (–1, 1, 0)

$\Rightarrow$ a – c = 0 and –a + b = 0

$\Rightarrow$ a = b = c

$\therefore$ Equation of plane is x – 1 + y – 1 + z – 1 = 0

or $\frac{x}{3}+\frac{y}{3}+\frac{z}{3}=1$. Its intercept on coordinate axes are

A(3, 0, 0), B(0, 3, 0), C(0, 0, 3)

Hence, the volume of tetrahedron OABC

$$= \frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}] = \frac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \frac{27}{6} = \frac{9}{2} \text{cu units}$$

Sol 7: Equation of plane containing the lines

$2x-y+z-3=0$ and $3x+y+z=5$ is

$(2x-y+z-3)+\lambda(3x+y+z-5)=0$

$\Rightarrow\ (2+3\lambda)x+(\lambda-1)y+(\lambda+1)z-3-5\lambda=0$

Since, distance of plane from (2, 1, –1) to above plane is $1/\sqrt{6}$.

$\therefore \left|\dfrac{6\lambda+4+\lambda-1-\lambda-1-3-5\lambda}{\sqrt{(3\lambda+2)^2+(\lambda-1)^2+(\lambda+1)^2}}\right|=\dfrac{1}{\sqrt{6}}$

$\Rightarrow 6(\lambda-1)^2=11\lambda^2+12\lambda+6$

$\Rightarrow \lambda=0,-\dfrac{24}{5}$

$\therefore$ Equations of planes are $2x-y+z-3=0$

and $62x+29y+19z-105=0$

Sol 8: (C) The line $\dfrac{x-3}{2}=\dfrac{y+2}{-1}=\dfrac{z+4}{3}$ lies in the plane,

then point $(3,-2,-4)$ lies on the plane

$\Rightarrow 3\ell-2m=5$ ...(i)

And line is $\perp$ to normal of plane

$\Rightarrow 2\ell-m=3$ ...(ii)

From (i) and (ii)

$\ell=1$ and $m=-1$

$\Rightarrow \ell^2+m^2=1^2+(-1)^2=2$

Sol 9: (A) The eq of line passes through $(1,-5,9)$ along $x=y=z$ is

$\dfrac{x-1}{1}=\dfrac{y+5}{1}=\dfrac{z-9}{1}=r$

The point on line $(r+1, r-5, r+9)$

This point also lies on the given plane

$r+1-r+5+r+9=5$

$r=-10$

The point in $(-9,-15,-1)$

Distance between $(1,-5,9)$ and $(-9,-15,-1)$

$=\sqrt{10^2+(-10)^2+(10)^2}=10\sqrt{3}$ unit

Sol 10: (C) $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{12}=r$

The point of interstation $(3r+2, 4r-1, 12r+2)$

Lies on plane, then

$3r+2-4r+1+12r+2-16=0$

$\Rightarrow 11r-11=0$

$\Rightarrow r=1$

The point in $(5,3,14)$

Distance $=\sqrt{(5-1)^2+(3-0)^2+(14-2)^2}$

$=\sqrt{16+9+144}$

$=\sqrt{169}=13$

Sol 11: (B) Let the two lines in a same plane interest at $P(x,y,0)$, then $2x-5y=3$ and $x+y=5$

On solving, we get $P\equiv(4,1,0)$

Any plane || to $x+3y+6z=1$ is

$x+3y+6z=\lambda$

$P(4,1,0)$ must satisfies it, then

$4+3+0=\lambda \Rightarrow \lambda=7$

The eq. to required plane

$\Rightarrow x+3y+6z=7$

Sol 12: (B) The parallel planes $2x+y+2z=8$

and $4x+2y+4z=-5$

Distance $=\dfrac{|-8\times 2-5|}{\sqrt{16+4+16}}=\dfrac{21}{\sqrt{36}}=\dfrac{21}{6}=\dfrac{7}{2}$

Sol 13: (A) Image of point $(1,3,4)$ is

$\dfrac{x-1}{2}=\dfrac{y-3}{-1}=\dfrac{z-4}{1}=\dfrac{-2(2-3+4+3)}{4+1+1}=-2$

$\Rightarrow(-3,5,2)$

Since line is parallel to plane direction, ratio will not change

Eq. of imaged line $\dfrac{x+3}{3}=\dfrac{y-5}{1}=\dfrac{z-2}{1}$

Sol 14: (A) $\ell+m+n=0 \Rightarrow n=-(\ell+m)$

Substituting in $\ell^2=m^2+n^2$

$\ell^2=m^2+(\ell+m)^2$

$\Rightarrow \ell^2=m^2+\ell^2+m^2+2m$

$\Rightarrow 2m^2+2m=0$

$\Rightarrow 2m(m+1)=0$

$\Rightarrow m=0,1$

if $m = 0, \ell = \frac{-1}{\sqrt{2}}, n = \frac{1}{\sqrt{2}}$

if $m = 1, \ell = 0 = n$ (not possible)

Therefore direction cosine

$$\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \text{or} \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$\cos\phi = \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) + (0)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)(0) = \frac{1}{2}$$

$$\Rightarrow \phi = \frac{\pi}{3}$$

Sol 15: (B) The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and

$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanars, then

$$\begin{vmatrix} 1 & 1 & -k \\ k & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow k(k+3) = 0$$

$$\Rightarrow K = 0, -3$$

Two values exist.

Sol 16: (A) Eq. of plane parallel to $x - 2y + 2z - 5 = 0$ is

$x - 2y + 2z = \lambda$

$\perp$ distance from origin is 1,

then $\frac{|0 - 0 + 0 - \lambda|}{\sqrt{1+4+4}} = 1 \Rightarrow \frac{|\lambda|}{3} = 1 \Rightarrow \lambda = \pm 3$

Eq. of plane $x - 2y + 2z = \pm 3$

Sol 17: (D) $\sin\theta = \frac{1+4+3\lambda}{\sqrt{1+4+9}\sqrt{1+4+\lambda}}$

$= \frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda}}$ ... (i)

Given $\cos\theta = \sqrt{\frac{5}{14}}$

$$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{5}{14}} = \frac{3}{\sqrt{14}}$$

From (i) $\frac{3}{\sqrt{14}} = \frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda}}$

$$\Rightarrow 3\sqrt{5+\lambda^2} = (5+3\lambda) \Rightarrow 9(5+\lambda^2) = 25 + 9\lambda^2 + 30\lambda$$

$$\Rightarrow 30\lambda = 20 \Rightarrow \lambda = \frac{2}{3}$$

Sol 18: (B) Statement-I: Since mid point of A(1, 0, 7) and B(1, 6, 3) is which lies on the line, therefore point B is image of A about line

Statement-II: Since it given that the line only bisects the line joining A and B, therefore not the correct explanation.

$$\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \text{or} \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$\cos\theta = \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right) + (0)\left(\frac{1}{\sqrt{2}}\right) + (0)$$

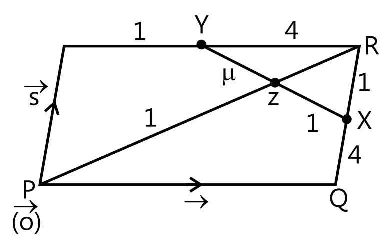
$$= \frac{1}{2} \Rightarrow 9 = \frac{\pi}{3}$$

JEE Advanced/Boards

Exercise 1

Sol 1: Let point P be taken as origin and $\vec{q}, \vec{s}$ are the position vectors of Q and S points respectively.

$\Rightarrow \overrightarrow{PR} = \vec{q} + \vec{s}$



$$\text{P.V. of } X = \frac{\vec{q}+4(\vec{q}+\vec{s})}{5} = \frac{5\vec{q}+4\vec{s}}{5}$$

$$\text{P.V. of } Y = \frac{4\vec{s}+\vec{q}+\vec{s}}{5} = \frac{\vec{q}+5\vec{s}}{5}$$

Let, $\frac{PZ}{ZR} = \frac{1}{\lambda}$ and $\frac{YZ}{ZX} = \mu$

$$\text{P.V. of } P = \frac{\vec{q}+\vec{s}}{\lambda+1} = \frac{\mu\left(\vec{q}+\frac{4}{5}\vec{s}\right)+\left(\frac{\vec{q}}{5}+\vec{s}\right)}{\mu+1}$$

$$\Rightarrow \frac{1}{\lambda+1} = \frac{\mu+\frac{1}{5}}{\mu+1} \quad \text{... (i)}$$

$$\Rightarrow \frac{1}{\lambda+1} = \frac{\mu+\frac{1}{5}}{\mu+1} \quad \text{... (ii)}$$

From (i) & (ii), we get

$\mu = 4, \lambda = \frac{4}{21}$

$\Rightarrow \frac{PZ}{ZR} = \frac{21}{4} \Rightarrow \frac{PZ}{PR} = \frac{21}{25}$

Sol 2: p (x ,y)

$\overrightarrow{PA} \bullet \overrightarrow{PB} + 3\overrightarrow{OA} \bullet \overrightarrow{OB} = 0$

$(x-1)(x+1) + y^2 + 3(-1) = 0$

$x^2 + y^2 = 4$

$|\overrightarrow{PA}| \cdot |\overrightarrow{PB}|$

$\sqrt{(x-1)^2 + y^2} \quad \sqrt{(x+1)^2 + y^2}$

$\sqrt{(5-2x)}\sqrt{(5+2x)} = \sqrt{25-4x^2}$

Max is $\sqrt{25} = 5 = M$

Min = $\sqrt{9} = 3$

$M^2 + m^2 = 34$

Sol 3: $|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{a}+\vec{b}| = 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 1;\ \vec{a}\cdot\vec{b} = \frac{-1}{2}$

$\theta = 120°$

$\angle\left(2\vec{a}+\vec{b}\ \&\ \vec{b}\right)$

$\Rightarrow$ (2a + b) (b) = |2a + b| |b| cos θ_1

$\Rightarrow 2a\ .\ b + |b|^2 = \sqrt{4a^2 + b^2 + 4a\ .\ b}\ \ |b|\ \cos\theta_1$

$\Rightarrow -1+1 = \cos\theta_1 \times k$

$\Rightarrow \cos\theta_1 = 0$

$\Rightarrow \theta_1 = \frac{\pi}{2}$

Sol 4: $c = \lambda\vec{a} + \mu\vec{b}$

$|c|^2 = \lambda^2 + \mu^2 + 2\lambda\mu\vec{a}\ .\ \vec{b} = 1$

$\Rightarrow \lambda^2 + \mu^2 - \lambda\mu = 1$

$\Rightarrow \vec{c}.\vec{a} = \lambda + \mu\ (\vec{a}\ .\ \vec{b}) = 0$

$\Rightarrow \lambda - \frac{\mu}{2} = 0$

$\Rightarrow \lambda = \frac{\mu}{2} \Rightarrow u = 2\lambda$

$\Rightarrow \lambda^2 + 4\lambda^2 - 2\lambda^2 = 1$

$\Rightarrow \lambda = \frac{1}{\sqrt{3}}, \mu = \frac{2}{\sqrt{3}} \Rightarrow \left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

Sol 5: $\vec{P} = x\vec{a} + y\vec{c} \Rightarrow \vec{p} = y\vec{c}$

$0 \le \vec{p}.\vec{a} = x \le 1\ x \in [0,1]$

$0 \le \vec{p}.\vec{b} = x\vec{a}.\vec{b} \le 1\ x \in [-2,0] \Rightarrow x = 0$

$\vec{p}.\vec{c} = y$

$\vec{c} = \frac{\vec{a}}{\sqrt{3}} + \frac{2\vec{b}}{\sqrt{3}}$

$\vec{p}.\vec{c} = \frac{\vec{p}.\vec{a}}{\sqrt{3}} + \frac{2\vec{p}.\vec{b}}{\sqrt{3}}$

$\vec{p} = x\vec{a} + y\left(\frac{\vec{a}+2\vec{b}}{\sqrt{3}}\right)$

Sol 6: For max. x and y; $x + \frac{y}{\sqrt{3}} = \frac{2y}{\sqrt{3}} \Rightarrow y = \sqrt{3}x$

x = 1; y = $\sqrt{3}$

$\Rightarrow \vec{p} = 2\vec{a} + 2\vec{b}$

Sol 7: The coordinates of the points, O and P, are (0, 0, 0) and (1, 2, -3) respectively.

Therefore, the direction ratios of OP are (1 - 0) = 1, (2 - 0) = 2 and (-3-0) = -3

It is knows that the equation of the plane passing through the point $(x_1\ y_1\ z_1)$ is

$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$, where a, b and c are the direction ratio of normal.

Here, the direction ratios of normal are 1, 2 and -3 and the point P is (1, 2, -3).

Thus, the equation of the required plane is

$1(x-1) + 2(y-2) - 3(z+3) = 0$

$\Rightarrow x + 2y - 3z - 14 = 0$

Sol 8: (i) $\vec{A} = [x_1y_1z_1]; \vec{B} = [x_2y_2z_2]; C = [x_3y_3z_3]$

$\vec{A}\left(\vec{B}\times\vec{C}\right) = 0$ all are coplanar

$\vec{A}\times\vec{B} = 0 = \vec{B}\times\vec{C} = \vec{C}\times\vec{A}$ i.e. all are mutually $\perp$ which simultaneously is not possible.

(ii) P = (x_1, y_2, x_3) Q = $(y_1y_2y_3)$ O (0,0,0)

In ΔPOQ; OP = $x_1i + x_2j + x_3k$

OQ = $y_1i + y_2j + y_3k$

OP . OQ = $x_1y_1 + x_2y_2 + x_3y_3\ x_1 > 0\ y_1 > 0$

OP . OQ < 0 [i.e. it can never be zero]

Sol 9: $A = (-5,22,5)$; $B = (1,2,3)$; $C = (4,3,2)$

$D = (-1,2,-3)$

and $\Delta AEF = \sqrt{S}$

$\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{DB}$

$\overrightarrow{OE} - \overrightarrow{OD} = \overrightarrow{OA} - \overrightarrow{OD} + \overrightarrow{OB} - \overrightarrow{OD}$

$\overrightarrow{OE} = -5i + 22j + 5k + i + 2j + 3k - (-i + 2j - 3k)$

$\overrightarrow{OE} = -3i + 22j + 11k$

$\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{BC}$

$\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{OF} = -5i + 22j + 5k + 4i + 3j + 2k - i - 2j - 3k$

$OF = -2i + 23j + 4k$

Area $= \frac{1}{2}\left[\overrightarrow{AE} \times \overrightarrow{AF}\right] = \frac{1}{2}\left[(2i + 6k) \times (3i + j - k)\right]$

$$\frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 6 \\ 3 & 1 & -1 \end{vmatrix} = \frac{1}{2}\hat{i}(-6) - \hat{j}(-2-18) + \hat{k}(2) = -6\hat{i} + 20\hat{j} + 2\hat{k}$$

$$= \frac{1}{2}\sqrt{36 + 400 + 4} = \sqrt{\frac{440}{4}} = \sqrt{110} \Rightarrow S = \sqrt{110}$$

Sol 10: $\left((a-2)\alpha^2 + (b-3)\alpha + c\right)x +$

$\Rightarrow \left((a-2)\beta^2 + (b-3)\beta + c\right)y +$

$\Rightarrow \left((a-2)\gamma^2 + (b-3)\gamma + c\right)\left(x \times y\right) = 0$

$\Rightarrow a - 2 = b - 3 = c = 0$

$\Rightarrow a = 2$; $b = 3$; $c = 0$

$\Rightarrow a^2 + b^2 + c^2 = 13$

Sol 11: The equation of the given line is

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ ... (i)

The equation of the given plane is

$\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$ ... (ii)

Substituting the value of from equation (i) in equation (ii), we obtain.

$$\left[2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\right]\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow \left[(3\lambda + 2)\,\hat{i} + (4\lambda - 1)\hat{j}(2\lambda + 2)\,\hat{k}\right].\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$\Rightarrow \lambda = 0$$

Substituting this value in equation (i), we obtain the equation of the line as $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates (2, -1, 2). The point is (-1, -5, -10).

The distance d between the points, (2, -1, 2) and (-1, -5, -10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$$
$$= \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

Sol 12: $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z-3}{c}$

Parallel to the plane $x + 5y + 4z = 0$

$\Rightarrow a + 5b + 4c = 0$

$\Rightarrow (-1 + 2\lambda, 2 + \lambda, -4 + 2\lambda) = \Rightarrow (1 + ka, 2 + kb, 3 + kc)$

$\Rightarrow \frac{ka + 2}{2} = \frac{2kb}{2} = \frac{7 + kc}{2}$

$\Rightarrow \frac{2}{2b - a} = \frac{7}{2b - c} = \frac{5}{a - c}$

$\Rightarrow 10b = 7a - 2c$ ....(i)

$\Rightarrow \frac{7a - 10b}{2} = \frac{-a - 5b}{4}$

$\Rightarrow a = b$; $c = \frac{-3a}{2}$ $a = 2b = 2c = -3$

Sol 13: $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \lambda$

$\lambda_a = 3 + 2k$; $\lambda_b = 3 + k$; $\lambda_c = k$

$$\frac{1}{2} = \left|\frac{2a + b + c}{\sqrt{6\sqrt{a^2 + b^2 + c^2}}}\right| \quad 3\,(a^2 + b^2 + c^2) = 2(2a + b - c)^2$$

$$\frac{3 + 2k}{a} = \frac{3 + k}{b} = \frac{k}{c}$$

$$\frac{3a - 3b}{2b - a} = \frac{3c}{b - c} = \frac{3c}{a - 2c}$$

$\Rightarrow$ a = 1, b = 2, c = –1 or a = –1 b = 1, c = –2

Exercise 2

Single Correct Choice Type

Sol 1: (B) Direction cosines of PQ (2, 3, –6) (3, –4, 5)

Ratios = 2 – 3, 3 + 4, –6 – 5 = –1, 7, –11

Direction cosines = $\frac{-1}{\sqrt{171}}, \frac{7}{\sqrt{171}}, \frac{-11}{\sqrt{171}}$

or $\frac{1}{\sqrt{171}}, \frac{-7}{\sqrt{171}}, \frac{11}{\sqrt{171}}$

Sol 2: (A) $\alpha = \frac{m+3}{m+1}$, $\beta = \frac{4m+1}{m+1}$, $\gamma = \frac{-6m-5}{m+1}$

As $\alpha = 0 \Rightarrow$ m = –3[A]

Sol 3: (D) 3x + 2y + z + 5 = x + y – 2z – 3

2x – y – λz = 7x + 10y – 8z are $\perp$ to each other

1. $\begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$ = i(–5) – j (–6 –1) + k(3–2) = –5i + 7j + k

2. $\begin{vmatrix} i & j & k \\ 2 & -1 & -\lambda \\ 7 & +10 & -8 \end{vmatrix}$ = i $(8+10\lambda)$ –j (–16 + 7λ) + k (+20

+ 7) –40 – 50λ + 112 – 49λ – 127 = 0$\Rightarrow$ λ = 1

Sol 4: (A) (2,4,5) —m— (a,b,g) —1— (3,5,7)

$\alpha = \frac{3m+2}{m+1} = 0 \Rightarrow$ m = $\frac{-2}{3}$

Sol 5: (B) $\cos\alpha = \frac{1}{\sqrt{3}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$

Sol 6: (A) $\alpha(x-a) + \beta(y-b) + \gamma(z-c) = 0$

$A\left(\frac{\beta b + \gamma c + \alpha a}{\alpha}, 0, 0\right)$

$(h,k,l) = \left(\frac{\alpha a + \beta b + \gamma c}{\alpha}, \frac{\alpha a + \beta b + \gamma c}{\beta}, \frac{\alpha a + \beta b + \gamma c}{\gamma}\right)$

(h – a)α = $\beta b + \gamma c$

(k – b)β = $\alpha a + \gamma c$

$(l - c)$ γ = $\alpha a + \beta b$

$\begin{vmatrix} h-a & -b & -c \\ -a & k-b & -c \\ -a & -b & l-c \end{vmatrix} = 0$

$(h-\alpha)\left[(k-b)(l-c) - bc\right] + b\left[-al + ac - ac\right]$ – c [ab + ak – ab] = 0

$(h-\alpha)$ $[kl - kc - bl]$ $-bal - cak = 0$

$hkl - hkc - hbl - akl + akc + abl - bal - cak = 0$

$ayz + bzx + cxy = xyz$

Sol 7: (A) $\frac{2x - y + 2z + 3}{3} = -\frac{(3x - 2y + 6z + 8)}{7}$

$p_1p_2 + q_1q_2 + r_1r_2 = 6 + 2 + 12 > 0$

+ve $\rightarrow$ acute

23x – 13y + 32z + 45 = 0[C]

Sol 8: (C) $\frac{x - 4/3}{2} = \frac{y + 6/5}{3} = \frac{z - 3/2}{4} = \lambda$

$\left(\frac{4}{3} + 2\lambda, \frac{-6}{5} + 3\lambda, \frac{3}{2} + 4\lambda\right)$

$\left(\frac{5}{3}, \frac{8}{5}, \frac{9}{2}\right) = \left(\frac{4}{3} + 5\lambda, \frac{-6}{5} + 8\lambda, \frac{3}{2} + 9\lambda\right)$

Both passes through $\left(\frac{+4}{3}, \frac{-6}{5}, \frac{3}{2}\right)$

Minimum distance is zero.

Sol 9: (C) $\frac{x}{2} + \frac{y}{\beta} = \frac{z}{\gamma}$

$\Rightarrow \alpha(b + c) + \beta(a + c) + \gamma(a + b) = 0$

$\Rightarrow \alpha(b - c) + \beta(c - a) + \gamma(a - b) = 0$

$\Rightarrow \alpha b + \beta c + \gamma a = 0$; $\alpha c + \beta a + \gamma b = 0$

$\Rightarrow \alpha = a^2 - bc$

$\Rightarrow \beta = b^2 - ac$

$\Rightarrow \gamma = c^2 - ab$

$\Rightarrow \frac{x}{a^2 - bc} = \frac{\gamma}{b^2 - ac} = \frac{z}{c^2 - ab}$

Assertion Reasoning Type

Sol 10: (C) y + z + 1 = 0 [0, 1, 1]

x-axis [1, 0, 0]

$\sin\theta = 0$

R is wrong.

Previous Years' Questions

Sol 1: (D) Let the equation of plane be

$a(x-1) + b(y+2) + c(z-1) = 0$ which is perpendicular to $2x - 2y + z = 0$ and

$x - y + 2z = 4.$

$\Rightarrow 2a - 2b + c = 0$ and $a - b + 2c = 0$

$$\Rightarrow \frac{a}{-3} = \frac{b}{-3} = \frac{c}{0} \Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{0}.$$

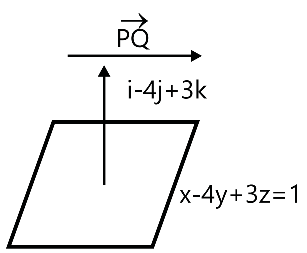
So, the equation of plane is

$x - 1 + y + 2 = 0$ or $x + y + 1 = 0$

Its distance from the point (1, 2, 2) is $\frac{|1+2+1|}{\sqrt{2}} = 2\sqrt{2}.$

Sol 2: (A) Given $\overrightarrow{OQ} - (1-3\mu)\hat{i} + (\mu - 1)\hat{j} + (5\mu + 2)\hat{k}$,

$\overrightarrow{OP} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ (Where O is origin)



Now, $\overrightarrow{PQ} = (1 - 3\mu - 3)\hat{i} + (\mu - 1 - 2)\hat{j} + (5\mu + 2 - 6)\hat{k}$

$= (-2 - 3\mu)\hat{i} + (\mu + 3)\hat{j} + (5\mu - 4)\hat{k}$

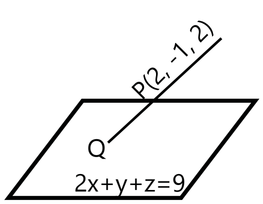
$\because$ $\overrightarrow{PQ}$ is parallel to the plane

$x - 4y + 3z = 1$

$\therefore$ $-2 - 3\mu - 4\mu + 12 + 15\mu - 12 = 0$

$\Rightarrow$ $8\mu = 2 \Rightarrow \mu = \frac{1}{4}$

Sol 3: (C) Since, $\ell = m = n = \frac{1}{\sqrt{3}}$



$\therefore$ Equation of line are $\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}}$

$\Rightarrow x - 2 = y + 1 = z - 2 = r$

$\therefore$ Any point on the line is $Q \equiv (r+2, r-1, r+2)$

$\because$ Q lies on the plane $2x + y + z = 9$

$\Rightarrow$ $4r + 5 = 9 \Rightarrow r = 1$

$2(x+2) + (r-1) + (r+2) = 9$

$\therefore$ Q(3,0,3)

$\therefore$ $PQ = \sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2} = \sqrt{3}$

Sol 4: (D) Given planes are $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ For $z = 0$, we get $x = 3$,

$y = -1$

Direction ratios of planes are <3, –6, –2> and <2, 1, –2>

then the DR's of line of intersection of planes is < 14, 2, 15> and line is

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z-0}{15} = \lambda \text{ (say)}$$

$\Rightarrow$ $x = 14\lambda + 3, y = 2\lambda - 1, z = 15\lambda$

Hence, statement I is false.

But statement II is true.

Sol 5: (D) Given three planes are

$P_1 : x - y + z = 1$ ... (i)

$P_2 : x + y - z = -1$ ... (ii)

and $P_3 : x - 3y + 3z = 2$ ... (iii)

Solving Eqs. (i) and (ii), we have $x = 0, z = 1 + y$

which does not satisfy Eq. (iii)

As, $x - 3y + 3z = 0 - 3y + 3(1 + y) = 3(\neq 2)$

$\therefore$ Statement-II is true.

Next, since we know that direction ratio's of line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$

and $a_2x + b_2y + c_2z + d_2 = 0$ is

$b_1c_2 - b_2c_1, c_1a_2 - a_1c_2, a_1b_2 - a_2b_1$

Using above result.

Direction ratio's of lines L_1, L_2 and L_3 are 0, 2, 2 ; 0; – 4, –4; 0, –2, –2

Respectively

$\Rightarrow$ All the three lines L_1, L_2, and L_3 are parallel pairwise.

$\therefore$ Statement-I is false.

Sol 6: (B) The equation of given lines in vector form may be written as

$L_1 : \vec{r} = (-\hat{i} - 2\hat{j} - \hat{k}) + \lambda(3\hat{i} + \hat{j} + 2\hat{k})$

and $L_2 : \vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k})$

Since, the vector perpendicular to both L_1 and L_2.

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$$

$\therefore$ Required unit vector

$$= \frac{(-\hat{i} - 7\hat{j} + 5\hat{k})}{\sqrt{(-1)^2 + (-7)^2 + (5)^2}} = \frac{1}{5\sqrt{3}}(-\hat{i} - 7\hat{j} + 5\hat{k})$$

Sol 7: (D) The shortest distance between L_1 and L_2 is

$$\left| \frac{((2-(-1))\hat{i} + (2-2)\hat{j} + (3-(-1))\hat{k}) \cdot (-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} \right|$$

$$= \left| \frac{(3\hat{i} + 4\hat{k}) \cdot (-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} \right| = \frac{17}{5\sqrt{3}} \text{unit.}$$

Sol 8: (C) The equation of the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the given lines L_1 and L_2 may be written as

$$(x+1) + 7(y+2) - 5(z+1) = 0$$

$$\Rightarrow x + 7y - 5z + 10 = 0$$

$$= \left| \frac{1 + 7 - 5 + 10}{\sqrt{1 + 49 + 25}} \right| = \frac{13}{\sqrt{75}} \text{units.}$$

Match the Columns

Sol 9: $A \to r$; $B \to q$; $C \to p$; $D \to s$

$$\text{Let } \Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

(A) If $a + b + c \neq 0$ and $a^2 + b^2 + c^2$

$\Rightarrow \Delta = 0$ and $a = b = c \neq 0$

The equations represent identical planes.

(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$\Rightarrow \Delta = 0$

$\Rightarrow$ the equations have infinitely many solutions.

$ax + by = (a+b)z$

$bx + cy = (b+c)z$

$\Rightarrow (b^2 - ac)y = (b^2 - ac)z \Rightarrow y = z$

$\Rightarrow ax + by + cy = 0$

$\Rightarrow ax = ay$

$\Rightarrow x = y = z.$

(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$\Rightarrow \Delta \neq 0$

$\Rightarrow$ The equation represent planes meeting at only one point.

(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow a = b = c = 0$

$\Rightarrow$ The equations represent whole of the three dimensional space.

Sol 10: (i) Equations of a plane passing through (2, 1, 0) is

$a(x-2)+b(y-1)+c(z)=0$

It also passes through (5, 0, 1) and (4, 1, 1)

$3a - b + c = 0$ and $2a + 0b + c = 0$

On solving, we get $\frac{a}{-1} = \frac{b}{-1} = \frac{c}{2}$

$\therefore$ Equation of plane is

$-(x-2) - (y-1) + 2(z-0) = 0$

$\Rightarrow -x + 2 - y + 1 + 2z = 0$

$\Rightarrow x + y - 2z = 3$

(ii) Let the coordinate of Q (α, β, γ)

Equation of line PQ $= \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2}$

Since, mid point of P and Q is $\left(\frac{\alpha+2}{2}, \frac{\beta+1}{2}, \frac{\gamma+6}{2}\right)$.

Which lies in a line P

$$\Rightarrow \frac{\frac{\alpha+2}{2} - 2}{1} = \frac{\frac{\beta+1}{2} - 1}{1} = \frac{\frac{\gamma+6}{2} - 6}{-2}$$

$$= \frac{1\left(\frac{\alpha+2}{2} - 2\right) + 1\left(\frac{\beta+1}{2} - 1\right) - 2\left(\frac{\gamma+6}{2} - 6\right)}{1 \cdot 1 + 1 \cdot 1 + (-2)(-2)} = 2$$

$$\left\{\text{since}, \left(\frac{\alpha+2}{2}\right) + 1\left(\frac{\beta+1}{2}\right) - 2\left(\frac{\gamma+6}{2}\right) = 3\right\}$$

$\Rightarrow \alpha = 6, \beta = 5, \gamma = -2$

$\Rightarrow$ Q(6, 5, –2)

Sol 11: Let the equation of the plane ABCD be ax + by + cz + d = 0, the point A″ be (α,β,γ) and the height of the parallelepiped ABCD be h.

$$\Rightarrow \frac{|a\alpha+b\beta+c\gamma+d|}{\sqrt{a^2+b^2+c^2}} = 90\%h$$

$$\Rightarrow a\alpha+b\beta+c\gamma+d = \pm 0.9h\sqrt{a^2+b^2+c^2}$$

$\therefore$ Locus is, ax + by + cz + d = $\pm 0.9h\sqrt{a^2+b^2+c^2}$

$\therefore$ Locus of A″ is a plane parallel to the plane ABCD.

Sol 12: (B, C, D) According to given data, we have

$$P(3,0,0), Q(3,3,0), R(0,3,0), S\left(\frac{3}{2},\frac{3}{2},3\right)$$

$$\overrightarrow{OQ} = 3\hat{i}+3\hat{j}$$

$$\overrightarrow{OS} = \frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+3\hat{k}$$

$$\overrightarrow{OQ}.\overrightarrow{OS} = \left|\overrightarrow{OQ}\right|\left|\overrightarrow{OS}\right|\cos\phi$$

$$\frac{9}{2}+\frac{9}{2} = 9\sqrt{2}\times\frac{3\sqrt{3}}{\sqrt{2}}\cos\phi \Rightarrow 9 = 9\sqrt{3}\cos\phi$$

$$\Rightarrow \phi = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

The equation of plane containing ΔOQR is $x-y=0$

The $\perp$ distance of point (3, 0, 0) from the plane $x-y=0$ is given by

$$= \left|\frac{3-0}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

The equation of RS

Direction ratios of RS $< \frac{-3}{2}, \frac{3}{2}, -3 >$ or $< 1,-1,2 >$

Equation of line RS $\frac{x}{1} = \frac{y-3}{-1} = \frac{z}{2} = r$

$\Rightarrow$ point on line $(r, 3-r, 2r)$

$$r+(3-r)(-1)+2(2r) = 0 \quad \Rightarrow r-3+r+4r = 0$$

$$\Rightarrow r = \frac{1}{2} \Rightarrow \text{point}\left(\frac{1}{2},\frac{5}{2},1\right)$$

Perpendicular distance

$$= \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{5}{2}\right)^2+1} = \sqrt{\frac{1}{4}+\frac{25}{4}+1} = \sqrt{\frac{30}{4}} = \sqrt{\frac{15}{2}}$$

Sol 13: (C) Let $P^1(3, 1, 7)$

The imago of P′ given by

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = -\frac{2(3-1+7-3)}{3} = -4$$

$$\Rightarrow P(x, y, z) \equiv (-1, 5, 3)$$

Any plane passing through $P(-1,5,3)$ and containing line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$

$$\begin{vmatrix} x & y & z \\ -1 & 5 & 3 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$x(5-6)-y(-1-3)+z(-2-5) = 0$$

$$\Rightarrow x-4y+72 = 0$$

Sol 14: (B, D) Any plane passes through point of intersection of plane P_1 and P_2 is $x+z-1+\lambda y = 0$

Given:

$$\left|\frac{0+0-1+\lambda}{\sqrt{1+1+\lambda^2}}\right| = 1 \Rightarrow |\lambda-1| = \sqrt{\lambda^2+2} \Rightarrow \lambda = -\frac{1}{2}$$

$\Rightarrow P_3$ is $2x-y+2z = 2$

Now, distance of P_3 from (α,β,γ) is 2.

$$\Rightarrow \left|\frac{2\alpha-\beta+2\gamma-2}{\sqrt{4+4+1}}\right| = 2$$

$\Rightarrow 2\alpha-\beta+2\gamma = 8$ and $2\alpha-\beta+2\gamma = -4$

Sol 15: (A, B) Since all the points on L are at same distance from planes P_1 and P_2 implies that line L is parallel to line of intersection of P_1 and P_2.

Let direction ratio of line L be α, β, γ then

$\alpha+2\beta-\gamma = 0$ and $2\alpha-\beta+\gamma = 0$

$\Rightarrow \alpha:\beta:\gamma \equiv 1:-3:-5$

Eq .of line L passes through origin

$$\frac{x-0}{1} = \frac{y-0}{-3} = \frac{z-0}{-5} = r$$

Foot of perpendicular from origin to the plane $P_1 \equiv x+2y-z+1 = 0$ can be obtained as

$$\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{-1} = \frac{-(0+0-0+1)}{1+4+1} = \frac{-1}{6}$$

$\Rightarrow\left(\frac{-1}{6},\frac{-1}{3},\frac{1}{6}\right)$

Now equation of perpendicular from any point on L is

$$\frac{x+\frac{1}{6}}{1}=\frac{y+\frac{1}{3}}{-3}=\frac{z-\frac{1}{6}}{-5}=\lambda$$

Any point on line $\left(\lambda-\frac{1}{6},-3\lambda-\frac{1}{3},-5\lambda+\frac{1}{6}\right)$

Point $\left(0,\frac{-5}{6},\frac{-2}{3}\right)$ and $\left(\frac{-1}{6},\frac{-1}{3},\frac{1}{6}\right)$ satisfy the line.

Sol 16: (C) Given: $P\equiv(\lambda,\lambda,\lambda)$

$L_1\equiv\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{0}=m$

$L_2\equiv\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{0}=n$

$\Rightarrow Q\equiv(m,m,1)$

$\Rightarrow R\equiv(n,-n,-1)$

$\overrightarrow{PQ}=(\lambda-m)\hat{i}+(\lambda-m)\hat{j}+(\lambda-1)\hat{k}$

Since $\overrightarrow{PQ}$ is perpendicular to L_1

$\Rightarrow\lambda-m+\lambda-m+0=0\ \Rightarrow\lambda=m\ \Rightarrow Q(\lambda,\lambda,1)$

Similarly, $R=(0,0,-1)$

Now, $PQ\perp PR$

$\Rightarrow(\lambda-m).(\lambda-n)+(\lambda-m).(\lambda+n)+(\lambda-1).(\lambda+1)=0$

$\Rightarrow 0+0+(\lambda-1).(\lambda+1)=0\ \Rightarrow\lambda=\pm1$

Negotiating $\lambda=1$ became points p and Q will coincide.
$\lambda=-1$

Sol 17: (D) Let any point P on the line $\frac{x+2}{2}=\frac{y-1}{-1}=\frac{z}{3}$ be $(2\gamma-2,-\gamma-1,3\gamma)$

P lies on the plane $x+y+2=3$

$\Rightarrow 2\gamma-2(-\gamma-1)+3\gamma=3\ \Rightarrow 4\gamma=6\ \Rightarrow\gamma=\frac{3}{2}$

$P\equiv\left(1,\frac{-5}{2},\frac{9}{2}\right)$

Point $(-2,-1,0)$ lies on the line, the feet of perpendicular Q is given by

$\frac{x+2}{1}=\frac{y+1}{1}=\frac{z-0}{1}=-\frac{(-2-1+0-3)}{1^2+1^2+1^2}$

$\Rightarrow Q\equiv(0,1,2)$

Direction ratio of line PQ joining feets of perpendicular are $\left(1,\frac{-7}{2},\frac{5}{2}\right)$

Equation of PQ $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

Sol 18: (A, D) Given lines

$L_1\equiv\frac{x-5}{0}=\frac{y-0}{3-\alpha}=\frac{z-0}{-2}$

$L_2\equiv\frac{x-\alpha}{0}=\frac{y}{-1}=\frac{z}{2-\alpha}$

L_1 and L_2 will be co-planar, then

$$\begin{vmatrix}0 & 3-\alpha & 2\\ 0 & -1 & 2-\alpha\\ 5-\alpha & 0 & 0\end{vmatrix}=0$$

$\Rightarrow(5-\alpha)\left[(3-\alpha)(2-\alpha)+2\right]=0$

$\Rightarrow(5-\alpha)(\alpha-1)(\alpha-4)=0$

$\Rightarrow\alpha=1,4,5$

Sol 19: (A) $L_1\equiv\frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}=r_1$

$L_2\equiv\frac{x-y}{1}=\frac{y+3}{1}=\frac{z+3}{2}=r_2$

For point of intersection of L_1 and L_2

$2r_1+1=r_2+4\Rightarrow 2r_1-r_1=3$ ... (i)

$-r_1=r_2-3$ ... (ii)

and $r_1-3=2r_3-3$ ... (iii)

Form (i), (ii), (iii), we get $r_1=2,\ r_2=1$

The point of intersection $(5,-2,-1)$

Now, direction ratio of plane $\perp$ to P_1 and P_2 given by

$$\begin{vmatrix}i & j & k\\ 3 & 5 & -6\\ 7 & 1 & 2\end{vmatrix}$$

$=i(10+6-j(6+42)+k(3-35)$

$=16i-48j-32k$

Any plane passes through $(5,-2,-1)$ and having direction ratio of normal

$16(x-5)-48(y+2)-32(z+1)=0$

$\Rightarrow(x-5)-3(y+2)-2(z+1)=0$

$\Rightarrow x-3y-2z=13$

$\Rightarrow a=1,b=-3,c=-2$ and $d=13$

Sol 20: (A) Given: $Q(2,3,5)$ $R(1,-1,4)$

Direction ratio of line QR is $(1,4,1)$

The eq. of QR

$$\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-5}{1}=r$$

Any point on it $P(r+2,\ 4r+3,\ r+5)$

P lies on the plane $5x-4y-z=1$

$$5(r+2)-4(4r+3)-(r+5)=1$$

$$\Rightarrow 5r+10-16r-12-r-5=1$$

$$\Rightarrow r=-\frac{8}{12}=-\frac{2}{3}$$

$$\Rightarrow P\equiv\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$$

$$PT=\sqrt{\left(\frac{4}{3}-2\right)^2\left(\frac{1}{3}-1\right)^2+\left(\frac{13}{3}-4\right)^2}$$

Now, direction ration of PT is $(2,\ 2,-1)$

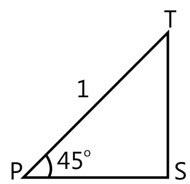
Angle between PT and QR

$$\cos\theta=\frac{1\times2+4\times2+1\times-1}{\sqrt{1+16+1}\sqrt{4+4+1}}$$

$$=\frac{9}{3\sqrt{2}\times3}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta=45^0$$

$$TS=PS=\frac{1}{\sqrt{2}}$$



Sol 21: (A) Eq of plane

$$x+2y+3z-2+k(x-y+z-3)=0$$

$$\Rightarrow x(1+k)+y(2-k)+z(3+k)-2-3k=0$$

Distend from point $(3,1,-1)$ is $\frac{2}{\sqrt{3}}$

$$\left|\frac{3(1+k)+1(2-k)-1(3+k)-(2+3k)}{\sqrt{(1+k)^2+(2-k)^2+(3+k)^2}}\right|$$

On solving, we get $k=\frac{-7}{3}$ $=\frac{2}{\sqrt{3}}$

Eq. of plane is $5x-11y+z=17$

Sol 22: (B, C) The lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$

and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$

are coplanar, then $\begin{vmatrix}2&0&0\\2&k&2\\5&2&k\end{vmatrix}=0$

$$2(k^2-4)=0\ \Rightarrow k=\pm2$$

For $k=2$, the lines are

$$\frac{x-1}{2}=\frac{y+1}{2}=\frac{z}{2}\ \text{ and }\ \frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{2}$$

Clearly plane $y+1=z$ contains both the lines

For $k=-2$, the lines are

$$\frac{x-1}{2}=\frac{y+1}{2}=\frac{z}{2}\ \text{ and }\ \frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{-2}$$

From options plane $y+z+1=0$ also contains both the lines.

Sol 23: Let the direction ratio of plane containing both the given lines are a, b, c then

$2a+3b+4c=0$ and $3a+4b+5c=0$

$$\Rightarrow \frac{a}{-1}=\frac{b}{2}=\frac{c}{-1}$$

Now, the equation of plane is

$$a(x-2)+b(y-3)+c(z-4)=0$$

$$\Rightarrow -(x-2)+2(y-3)-(z-4)=0$$

$$\Rightarrow -x+2+2y-6-z+4=0$$

$$\Rightarrow -x+2y-z=0$$

$$\Rightarrow x-2y+z=0$$

Distance between planes

$$\frac{|d-0|}{\sqrt{1+4+1}}=\sqrt{6}\ \Rightarrow |d|=6$$

Sol 24: (A) Distance of point $p(1,-2,1)$

From plane $x+2y-2z=\alpha$ is 5, then

$$\left|\frac{1+2x-2-2\times1-\alpha}{\sqrt{1+4+4}}\right|=5$$

$$|-5-\alpha|=5$$

$$\Rightarrow \alpha=10$$

For foot of perpendicular is M, then

$$\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\lambda$$

$M\equiv(\lambda+1,\ 2\lambda-2,-2\lambda+1)$ lies on plane

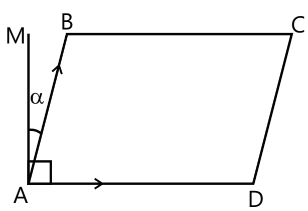
$$\Rightarrow \lambda+1+2(2\lambda-2)-2(2\lambda+1)-10=0$$

$$\Rightarrow \lambda+1+4\lambda-4+4\lambda-2-10=0$$

$$\Rightarrow 9\lambda=15$$

$$\Rightarrow \lambda=\frac{5}{3} \quad \Rightarrow Q\left(\frac{8}{3},\frac{4}{3},\frac{-7}{3}\right)$$

Sol 25: (B) Angle between $\overrightarrow{AB}$ and $\overrightarrow{AD}$



$$\cos\theta\frac{(2i+10j+11k).(-i+2j+2k)}{\sqrt{4+100+121}\sqrt{1+4+4}}=\frac{40}{15\times 3}=\frac{8}{9}$$

$$\Rightarrow \alpha=\frac{\pi}{2}-\cos-1\left(\frac{8}{9}\right)=\sin^{-1}\left(\frac{8}{9}\right)=\cos^{-1}\left(\frac{\sqrt{17}}{9}\right)$$

$$\Rightarrow \cos\alpha=\left(\frac{\sqrt{17}}{9}\right)$$