PROBLEM-SOLVING TACTICS

To verify SHM see whether force is directly proportional to y or see if $\frac{d^2x}{dt^2} + \omega^2 x = 0$ in cases when the equation is directly given compare with general equation to find the time period and other required answers

FORMULAE SHEET

1. Simple Harmonic Motion (SHM):

$F = -kx^n$

n is even - Motion of particle is not oscillatory

n is odd - Motion of particle is oscillatory.

If n = 1, F = -kx or F $\propto$ -x. The motion is simple harmonic.

x = 0 is called the mean position or the equilibrium position.

Condition for SHM $\frac{d^2x}{dt^2} \propto -x$

Acceleration, $a = \frac{F}{m} = -\frac{k}{m}x = -\omega^2 x$

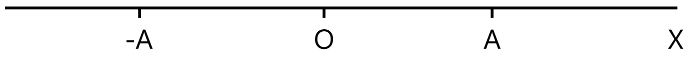

Figure 8.30

Displacement $x = A\cos\underbrace{(\omega t + \phi)}_{\text{phase angle}}$ (A is Amplitude)

Time period of SHM $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Frequency ν of SHM $\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Velocity of particle $v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$

Acceleration of particle $a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$

2. Energy in SHM:

Kinetic energy of particle $= \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}k\left(A^2 - x^2\right)$

Potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$

Total energy E= P.E + K.E = $\frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$

E is constant throughout the SHM.

3. Simple pendulum: Time period $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$

Here, ℓ is length of simple pendulum and $\vec{g}_{eff} = \vec{g} - \vec{a}$ where $\vec{g}$ is acceleration due to gravity and $\vec{a}$ is acceleration of the box or cabin etc. containing the simple pendulum.

4. Spring-block system: Time period $T = 2\pi\sqrt{\frac{m}{k}}$

5. Physical pendulum: Time period $T = 2\pi\sqrt{\frac{I}{mg\ell}}$

Here I is the moment of inertia about axis of rotation and ℓ is the distance of center of gravity from the point of suspension.

6. Torsional Pendulum:

$T = 2\pi\sqrt{\frac{I}{k}}$

I is the moment of Inertia about axis passing through wire, k is torsional constant of wire.

7. Springs in series and parallel

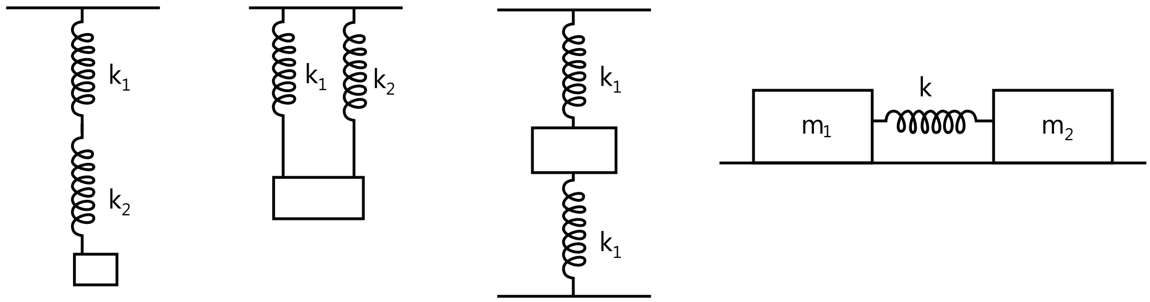

Figure 8.31

Series combination $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$

Parallel combination $k = k_1 + k_2$

8. For two blocks of masses m_1 and m_2 connected by a spring of constant k:

Time period $T=2\pi\sqrt{\frac{\mu}{k}}$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is reduced mass of the two-block system.