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PROBLEM-SOLVING TACTICS

Applying the principle of Conservation of Linear Momentum

(a) Decide which objects are included in the system.

(b) Relative to the system, identify the internal and external forces.

(c) Verify that the system is isolated.

(d) Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.

(e) Always check whether kinetic energy is conserved or not. If it is conserved, it gives you an extra equation. Otherwise use work-energy theorem, carefully.

(f) Try to involve yourself physically in the question, imagine various events. This would help in some problems where some parameters get excluded by conditions. This will also help in checking your answer.

Impulse

(g) Ignore any finite-value forces, while dealing with impulses.

(h) Write impulse equations carefully, because integration which we are unable to calculate will always cancel out.

Collisions

(i) Remembering special cases of collisions would be nice.

FORMULAE SHEET

Position of center of mass of a system: $\vec{r}_{com} = \dfrac{\sum_i m_i \vec{r}_i}{M}$

$$\vec{r}_{COM} = x_{COM}\hat{i} + y_{COM}\hat{j} + z_{COM}\hat{k}$$

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

For continuous bodies $x_{COM} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int x\,dm}{M}$

For a two-particle system, we have

$$r_1 = \left(\frac{m_2}{m_2 + m_1}\right)d \text{ and } r_2 = \left(\frac{m_1}{m_1 + m_2}\right)d$$

where d is the separation between the particles.

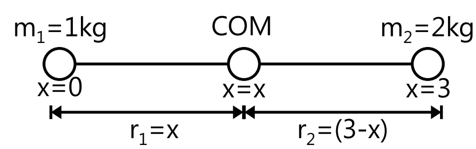

Figure 6.28

Center of Mass of a Uniform Rod $\left(\dfrac{L}{2}, 0, 0\right)$

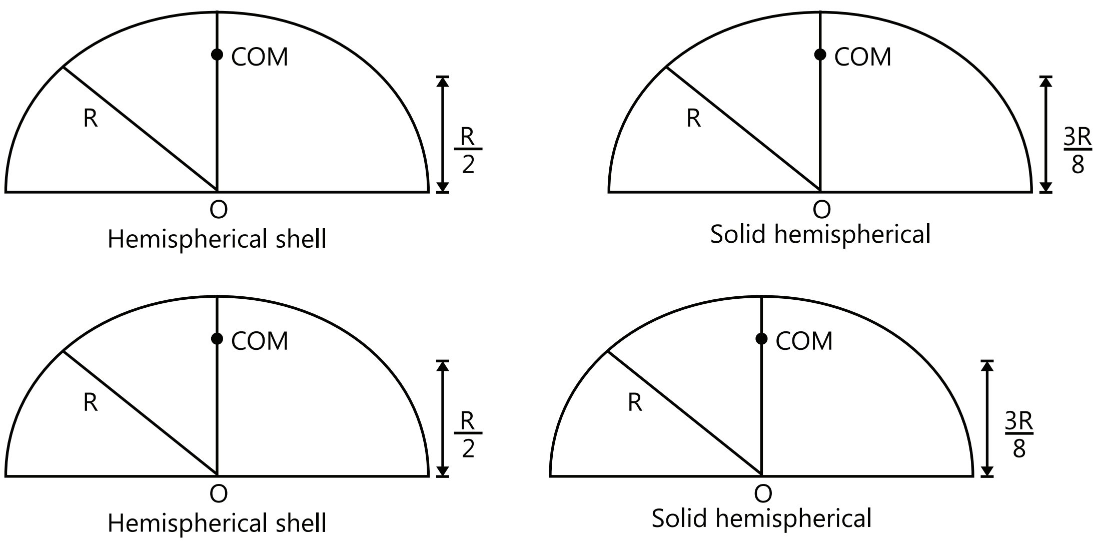

Figure 6.29

If some mass or area is removed from a rigid body, then the position of center of mass of the remaining portion is obtained from the following formula:

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2}$$

Where m_1 is the mass of the body after filling all cavities with same density and m_2 is the mass filled in the cavity. Cavity mass is assumed negative.

Velocity of COM $\vec{v}_{COM} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + ... + m_n} = \dfrac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$

Total momentum of a n-particle system $\vec{P}_{COM} = \vec{P}_1 + \vec{P}_2 + ... + \vec{P}_n = M\vec{v}_{COM}$

Acceleration of COM $\vec{a}_{COM} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n}{m_1 + m_2 + ... + m_n} \dfrac{\sum_i m_i \vec{a}_i}{\sum_i m_i}$

Net force acting on the system $\vec{F}_{COM} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n$

Net external force on center of mass is $M\vec{a}_{cm} = \vec{F}_{ext}$

If net force on the system $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = 0$ then, $\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + ... + \vec{P}_n$ = constant

Equation of motion of a body with variable mass is:

$$m\left(\frac{d\vec{v}}{dt}\right) = \vec{F} + \left(\frac{dm}{dt}\right)\vec{u}$$

Where $\vec{u}$ is the velocity of the mass being added(separated) relative to the given body of instantaneous mass m and $\vec{F}$ is the external force due to surrounding bodies or due to field of force.

In case of reducing mass of a system $\dfrac{dm}{dt} = \mu\ kgs^{-1}$

For a rocket we have, $m\left(\dfrac{d\vec{v}}{dt}\right) = m\vec{g} + \left(\dfrac{dm}{dt}\right)\vec{v}_r$

Where $\vec{v}_r$ is the velocity of the ejecting gases relative to the rocket.

In scalar form we can write

$$m\left(\frac{dv}{dt}\right) = -mg + v_r\left(-\frac{dm}{dt}\right)$$

Here $-\dfrac{dm}{dt}$ = rate at which mass is ejecting and $v_r\left(-\dfrac{dm}{dt}\right)$ =Thrust force.

Final velocity of rocket $v = u - gt + v_r \ln\left(\dfrac{m_0}{m}\right)$

Impulse of a force: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

Collision

(a) In the absence of any external force on the system the linear momentum of the system will remain conserved before, during and after collision, i.e.,

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v = m_1v'_1 + m_2v'_2 \qquad ...(i)$$

(b) In the absence of any dissipative forces, the mechanical energy of the system will also remain conserved, i.e.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \qquad ...(ii)$$

Head on Elastic Collision

$$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$v'_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

$$\frac{\text{separation speed after collision}}{\text{approach speed before collision}} = e$$

$$v'_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)v_2$$

$$v'_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)v_2 + \left(\frac{m_1 + em_1}{m_1 + m_2}\right)v_1$$

The C-frame: Total kinetic energy of system in K-frame is related to total kinetic energy in C-frame as:

$$K_{sys} = K_{sys/c} + \frac{Mv_c^2}{2}; \ M = \sum m_i$$

For a two-particle system: $\vec{P}_{1/c} = -\vec{P}_{2/c} = \frac{m_1 m_2}{m_1 + m_2}(\vec{v}_1 - \vec{v}_2)$

$$\text{Or } P_{1/c} = P_{2/c} = \mu v_{rel} = \frac{m_1 m_2}{m_1 + m_2}\left|\vec{v}_1 - \vec{v}_2\right| \text{ and } K_{sys/c} = \frac{\mu v_{rel}^2}{2} = \frac{P_{1/c}^2}{2}$$