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Class: 11
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Duration: 1 Hr 0 Min
Total Mark:40

About Examination: EXAM

Section 1 (Answer All the Questions) **40X1=40**

1) **Number of solutions of the equation $\tan^{-1}\frac{x}{1-x^2}+\tan^{-1}\frac{1}{x^3}=\frac{3\pi}{4}$ is**

Answer:
- 0

Solution:
Taking tan on both side

$$\frac{\frac{x}{1-x^2}+\frac{1}{x^3}}{1-\frac{1}{x^2\left(1-x^2\right)}}=-1 \Rightarrow x=1$$ But for x = 1 equation is not defined

2) **Which of the following is correct for all $x\in(-1,0)$**

Answer:
- $\cos^{-1}x=\cot^{-1}\frac{x}{\sqrt{1-x^2}}$

Solution:

$$x\in(-1,0)\Rightarrow \cos^{-1}x\in\left(\frac{\pi}{2},\pi\right),\ \tan^{-1}\frac{\sqrt{1-x^2}}{x}\in\left(\frac{-\pi}{2},0\right),\ \sin^{-1}\sqrt{1-x^2}\in\left(0,\frac{\pi}{2}\right)$$

$$\cot^{-1}\frac{x}{\sqrt{1-x^2}}\in\left(\frac{\pi}{2},\pi\right),\ \cos\left(\frac{\pi}{2}-\cot^{-1}x\right)=\sin\cot^{-1}x$$

3) **If $x\in[0,4\pi]$, $y\in[0,4\pi]$, then the number of ordered pairs (x, y) which satisfy the equation $\sin^{-1}\sin x+\cos^{-1}\cos y=\frac{3\pi}{2}$**

Answer:
- 4

Solution:
Solution : 4

4) ***The value of 'a' for which $ax^2+\sin^{-1}(x^2-2x+2)+\cos^{-1}(x^2-2x+2)=0$ has a real solution , is***

Answer:
- $-\frac{\pi}{2}$

Solution:
Here, $x^2-2x+2=(x-1)^2+1\geq 1$

But $-1\leq\left(x^2-2x+2\right)\leq 1$

Which is possible only when $x^2-2x+2=1$

$\therefore x=1$

Then, $a(1)^2+\sin^{-1}(1)+\cos^{-1}(1)=0$

$\Rightarrow a+\frac{\pi}{2}+0=0$

$\therefore a=-\frac{\pi}{2}$

5) ***The value of $\tan\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right)\right\}$ is***

Answer:
- $\frac{2}{3\sqrt{5}}$

Solution:

$$\tan\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right)\right\}=\tan\left\{\pi-\cos^{-1}\left(\frac{2}{7}\right)-\frac{\pi}{2}\right\}=\tan\left\{\frac{\pi}{2}-\cos^{-1}\left(\frac{2}{7}\right)\right\}=\tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\}=\tan\tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)=\frac{2}{3\sqrt{5}}$$

6) **The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is**

Answer:
- π

Solution:

$$\because \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)+\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)=\frac{2\pi}{3}+\left(\pi-\frac{2\pi}{3}\right)=\pi$$

7) **The sum to infinite terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\ldots\ldots+\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)+\ldots\ldots\ to\ \infty$ is**

Answer:
- $\frac{\pi}{4}$

Solution:

$$\lim n\to\infty\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)=\lim n\to\infty\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r.2^{r-1}}\right)=\tan^{-1}2^{\infty}-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

8) **The value of $\operatorname{cosec}^{-1}\sqrt{5}+\operatorname{cosec}^{-1}\sqrt{65}+\operatorname{cosec}^{-1}\sqrt{(325)}+\ldots\ldots\ to\ \infty$ is**

Answer:
- $\frac{\pi}{4}$

Solution:

$$\operatorname{cosec}^{-1}\sqrt{5}+\operatorname{cosec}^{-1}\sqrt{65}+\operatorname{cosec}^{-1}\sqrt{(325)}+\ldots\infty=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\ldots\infty=\lim n\to\infty\sum_{r=1}^{n}\cot^{-1}\left(2r^2\right)=\lim n\to\infty\sum_{r=1}^{n}\tan^{-1}\left(\frac{2}{4r^2}\right)=\lim n\to\infty\sum_{r=1}^{n}\tan^{-1}\left(\frac{}{1+4}\right.$$

9) **The value of $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)$ is**

Answer:
- $\frac{14}{15}$

Solution:

$$\sin\left(\tan^{-1}\frac{3}{4}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=\sin\left(\sin^{-1}\frac{3}{5}\right)+\cos\left(\cos^{-1}\frac{1}{3}\right)=\frac{3}{5}+\frac{1}{3}=\frac{14}{15}$$

10) The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

Answer:
- two

Solution: $x(x+1) \geq 0$ and $x^2 + x + 1 \leq 1 \Rightarrow x(x+1) \leq 0$

$\therefore x = 0 \text{or} -1$

11) Given$0 \leq x \leq \frac{1}{2}$ then the value of $\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right\} - \sin^{-1}x\right]$ is

Answer:
- 1

Solution: Put $x = \sin\theta \Rightarrow \tan\left[\sin^{-1}\left(\sin\left(\theta + \frac{\pi}{4}\right)\right) - \theta\right] = 1$

12) If the mapping f(x) = ax + b, a>0 maps [- 1,1] onto [0, 2] then $\cot\left[\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18\right]$ is equal to

Answer:
- f(1) + 1

Solution: $f(-1) = 0,\ f(1) = 2 \Rightarrow -a + b = 0$

$a + b = 2 \Rightarrow b = 1,\ a = 1 \Rightarrow f(x) = x + 1$

$$\cot\left[\text{Tan}^{-1}\frac{1}{7} + \text{Tan}^{-1}\frac{1}{8} + \text{Tan}^{-1}\frac{1}{18}\right] = \cot\left[\text{Tan}^{-1}\left(\frac{15}{55}\right) + \text{Tan}^{-1}\left(\frac{1}{18}\right)\right]$$

$$= \cot\left[\text{Tan}^{-1}\left(\frac{65}{195}\right)\right] = \cot\left[\text{Tan}^{-1}\left(\frac{1}{3}\right)\right] = 3 = f(2)$$

13) Number of solutions of the equation $\left(\tan^{-1}x\right)^2 + \left(\cot^{-1}x\right)^2 = \frac{5\pi^2}{8}$ is

Answer:
- 1

Solution: $\frac{\pi^2}{4} - 2\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{5\pi^2}{8} \Rightarrow \tan^{-1}x = \frac{-\pi}{4},\ \frac{3\pi}{4}$

14) The value of $\tan^2\left(\sec^{-1}3\right) + \cot^2\left(\text{cosec}^{-1}4\right)$ is

Answer:
- 23

Solution: $\sec^{-1}3 = \alpha,\ \text{cosec}^{-1}4 = \beta \Rightarrow \tan^2\alpha + \cot^2\beta = 9 - 1 + 16 - 1 = 23$

15) If $x = \sin\left(2\tan^{-1}2\right),\ y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$ then

Answer:
- $y^2 = 1 - x$

Solution: Let $\tan^{-1}2 = \alpha \Rightarrow x = \sin2\alpha = \frac{4}{5}$

$$y = \sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \frac{1}{\sqrt{5}}$$

16) The value of $\sin\left(\cot^{-1}(\cos(\tan^{-1}x))\right)$ is equal to

Answer:
- $\sqrt{\frac{x^2+1}{x^2+2}}$

Solution: $$\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+\cos^2\theta}}\right)\right) = \frac{1}{\sqrt{1+\cos^2\theta}} = \frac{\sec\theta}{\sqrt{1+\sec^2\theta}} = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

17) The value of $\cos^{-1}(\cos10)$ =

Answer:
- $4\pi - 10$

Solution: $\cos^{-1}(\cos10) = \cos^{-1}(\cos(4\pi - 10)) = 4\pi - 10$

18) If $\cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha} = x$ then sinx =

Answer:
- $\tan^2\left(\frac{\alpha}{2}\right)$

Solution: Let $\cot^{-1}\sqrt{\cos\alpha} = \theta \Rightarrow \sqrt{\cos\alpha} = \cot\theta$

$$\theta - \tan^{-1}(\cot\theta) = x \Rightarrow \theta - \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \theta\right)\right) = x$$

$$\Rightarrow 2\theta - \frac{\pi}{2} = x$$

$$\Rightarrow \sin x = -\cos2\theta = -\frac{1-\frac{1}{\cos\alpha}}{1+\frac{1}{\cos\alpha}} = \tan^2\left(\frac{\alpha}{2}\right)$$

19) The root of the equation $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$ is

Answer:
- $\frac{4}{3}$

Solution: $\frac{2x^2-1}{3x} = \frac{23}{36} \Rightarrow x = \frac{4}{3},\ \frac{-3}{8}$ but x can not be negative

20) If $\cos^{-1}x + \cos^{-1}y = 2\pi$, then $\sin^{-1}x + \sin^{-1}y$ is equal to

Answer:
- $-\pi$

Solution: conceptual

21) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3$ is equal to

Answer:
- $\frac{\pi}{4}$

Solution: conceptual

22) If $\cot^{-1}\alpha + \cot^{-1}\beta = \cot^{-1}x$, then $x =$

Answer:
- $\frac{\alpha\beta-1}{\alpha+\beta}$

Solution: conceptual

23) $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1}x$, then x=

Answer:
- $\frac{a+b}{1-ab}$

Solution: conceptual

24) $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$ is equal to

Answer:
- $\frac{2\pi}{3}$

Solution: conceptual

25) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} =$

Answer:
- $\frac{\pi}{4}$

Solution: conceptual

26) $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} =$

Answer:
- $\frac{\pi}{4}$

Solution: conceptual

27) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y =$

Answer:
- $\frac{\pi}{3}$

Solution: conceptual

28) $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then x=

Answer:
- 3

Solution: conceptual

29) $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) =$

Answer:
- $\frac{\pi}{4}$

Solution: conceptual

30) A solution of the equation $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ is

Answer:
- x=0

Solution: conceptual

31) If $x^2 + y^2 + z^2 = r^2$, then $\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan\left(\frac{zx}{yr}\right) =$

Answer:
- 0

Solution: conceptual

32) If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x equals

Answer:
- -1

Solution: conceptual

33) If $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$, then $x =$

Answer:
- None

Solution: conceptual

34) If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is

Answer:
- $\frac{1}{\sqrt{5}}$

Solution: conceptual

35) The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

Answer:
- 0

Solution: conceptual

36) If $4\sin^{-1}x + \cos^{-1}x = \pi$, then is equal to

Answer:
- 1/2

Solution: conceptual

37) If $\sin^{-1}\frac{3}{5} + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}C$, then $C =$

Answer:
- $\frac{56}{65}$

Solution: conceptual

38) $\sin\left\{\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$ is equal to

Answer:
- 1

Solution: conceptual

39) The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$ is

Answer:
- $\frac{\pi}{2}$

Solution: conceptual

40) The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)$ is

Answer:
- 30°

Solution: conceptual