## Exercise 1.1

### Question 1:

Use Euclid's division algorithm to find the HCF of:

**(i)** 135 and 225 **(ii)** 196 and 38220 **(iii)** 867 and 255

### Answer 1:

**(i)** 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

**(ii)** 196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

**(iii)** 867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore,

HCF of 867 and 255 is 51.

## Question 2:

Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where $q$ is some integer.

**Answer 2:**

Let $a$ be any positive integer and $b = 6$.

Then, by Euclid's algorithm, $a = 6q + r$ for some integer $q \geq 0$, and

$r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where $k_1$ is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where $k_2$ is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where $k_3$ is an integer

Clearly,

$6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where $k$ is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

## Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

**Answer 3:**

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

## Question 4:

Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer $m$.

[**Hint:** Let $x$ be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

**Answer 4:**

Let $a$ be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$ Or,

$$\begin{aligned} a^2 &= (3q)^2 \ or \ (3q+1)^2 \ or \ (3q+2)^2 \\ &= (3q)^2 \ or \ 9q^2 + 6q + 1 \ or \ 9q^2 + 12q + 4 \\ &= 3 \times (3q^2) \ or \ 3 \times (3q^2 + 2q) + 1 \ or \ 3 \times (3q^2 + 4q + 1) + 1 \\ &= 3k_1 \ or \ 3k_2 + 1 \ or \ 3k_3 + 1 \end{aligned}$$

Where $k_1$, $k_2$, and $k_3$ are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

## Question 5:

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

## Answer 5:

Let $a$ be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, every number can be represented as these three forms.

There are three cases.

**Case 1**: When $a = 3q$,

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$

Where $m$ is an integer such that $m = 3q^3$

**Case 2**: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where $m$ is an integer such that $m = (3q^3 + 3q^2 + q)$

**Case 3**: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where $m$ is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.